NC DAP
(NORTH CAROLINA DIAGNOSTIC ASSESSMENT AND PLACEMENT)

MATHEMATICS
STUDY GUIDE
(REVISED 11.18.15)

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NC DAP

MATHEMATICS

STUDY GUIDE
All students are encouraged to prepare for placement testing. Not reviewing for the placement tests could result in students being placed into courses below their actual skill level. This can delay student progress, so prepare as best you can!

The placement test will determine which English and math courses you will take when you attend Coastal. All of our courses are designed to help you succeed, so you will be in good hands, no matter where you place.

This study guide contains reminders, testing tips, an overview of the test, and sample questions.

The faculty and staff of Coastal Carolina Community College wish you the best of luck as you embark on your educational path. We look forward to working with you!

**Reminders**
- The test is computerized; you will be furnished scrap paper and pencil to make notes and/or calculations.
- Bring a photo ID and your student ID number on test day.
- With the exception of the essay (2 hour limit), each section is untimed.
- Once an answer is submitted, it cannot be changed.
- Unauthorized devices such as cell phones and iPads are not allowed.
- Work by yourself.

**Testing Tips**
- Get plenty of rest the night before you plan to take the test.
- Make sure you eat a good breakfast or lunch prior to testing.
- Take the test seriously; you may only test twice in a 12 month period.
- Don’t be discouraged; this test is designed to feel difficult.
- Write as much as you can for the essay; don’t just stop when you reach the required word count. Whatever you do, don’t skip this section!
Mathematics Overview
The NCCCS Diagnostic and Placement Mathematics test contains 72 questions that measure proficiency in six content areas. *This test is untimed.* The six content areas are as follows:

**Operations with Integers** — Topics covered in this category include:
- Problem events that require the use of integers and integer operations
- Basic exponents, square roots and order of operations
- Perimeter and area of rectangles and triangles
- Angle facts and the Pythagorean Theorem

**Fractions and Decimals** — Topics covered in this category include:
- Relationships between fractions and decimals
- Problem events that result in the use of fractions and decimals to find a solution
- Operations with fractions and decimals
- Circumference and area of circles
- The concept of $\pi$
- Application problems involving decimals

**Proportions, Ratios, Rates and Percentages** — Topics covered in this category include:
- Conceptual application problems containing ratios, rates, proportions and percentages
- Applications using U.S. customary and metric units of measurement
- Geometry of similar triangles

**Expressions, Linear Equations and Linear Inequalities** — Topics covered in this category include:
- Graphical and algebraic representations of linear expressions, equations and inequalities
- Application problems using linear equations and inequalities

**Graphs and Equations of Lines** — Topics covered in this category include:
- Graphical and algebraic representations of lines
- Interpretation of basic graphs (line, bar, circle, etc.)

**Polynomials and Quadratic Applications** — Topics in this category include:
- Graphical and algebraic representations of quadratics
- Finding algebraic solutions to contextual quadratic applications
- Polynomial operations
- Factoring polynomials
- Applying factoring to solve polynomial equations

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FRACTIONS

Method:

To simplify a fraction, divide the numerator and denominator by all common factors.

**Examples**

Ex: \[
\frac{12}{18} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}
\]

To multiply fractions:

1. Divide out all factors common to a numerator and any denominator.

Ex: \[
\frac{5}{6} \times \frac{4}{15} = \frac{15 \times 4}{6 \times 15} = \frac{2}{9}
\]

2. Multiply numerators.

3. Multiply denominators.

To divide fractions, invert the second fraction and multiply.

Ex: \[
\frac{5}{6} \div \frac{1}{2} = \frac{5}{6} \times \frac{2}{1} = \frac{5 \times 2}{6 \times 1} = \frac{5}{3}
\]

To add or subtract fractions:

1. Find the Least Common Denominator (LCD)

Ex: \[
\frac{1}{2} + \frac{3}{7} + \frac{5}{8} \quad \text{LCD}=56
\]

2. In each fraction, multiply the numerator and denominator by the same number to obtain the common denominator.

\[
\frac{1 \times 28}{2 \times 28} + \frac{3 \times 8}{7 \times 8} + \frac{5 \times 7}{8 \times 7}
\]

3. Add or subtract the numerators and keep the common denominator.

\[
\frac{28 + 24 + 35}{56} = \frac{87}{56}
\]

To change a mixed number to an improper fraction:

1. Multiply to the denominator by the whole number.

Ex: \[
5 \frac{2}{3} = \frac{3 \times 5 + 2}{3} = \frac{17}{3}
\]

2. Add the product to the numerator.

3. Place the sum over the denominator.
To change an improper fraction to a mixed number:

1. Divide the denominator into the numerator.
   \[ \frac{42}{5} = 8\frac{2}{5} \]  
   since \( 5 \left(\frac{42}{5}\right) = \frac{40}{2} \)

2. The whole number in the mixed number is the quotient, and the fraction is the remainder over the denominator.

To change a whole number to a fraction, write the number over 1.

\[ 9 = \frac{9}{1} \]

To multiply or divide the whole numbers and/or mixed numbers:

1. Change to improper fractions.
   \[ 2\frac{1}{7} \div 5 \]

2. Multiply or divide the fractions
   \[ \frac{7}{3} \div \frac{5}{1} = \frac{7}{3} \times \frac{1}{5} = \frac{7}{15} \]

3. Change the answer to mixed number.

To add mixed numbers, change to improper fractions, add, and then convert to a mixed number

\[ 6\frac{3}{4} + 2\frac{5}{8} \]

\[ = \frac{27}{4} + \frac{21}{8} = \frac{54}{8} + \frac{21}{8} = \frac{75}{8} = 9\frac{3}{8} \]

OR

Add the whole numbers and fractions separately.

\[ \frac{6\frac{3}{4}}{8\frac{11}{8}} + \frac{6\frac{5}{8}}{18\frac{10}{15}} = 8 + 1\frac{3}{8} = 9\frac{3}{8} \]

If an improper fraction results, change it to a mixed number and add the whole numbers.

To subtract mixed numbers, change to improper fractions, subtract, and convert to mixed numbers.

\[ 8\frac{1}{5} - 4\frac{2}{3} \]

\[ = \frac{41}{5} - \frac{14}{3} = \frac{123}{15} - \frac{70}{15} = \frac{53}{15} = 3\frac{8}{15} \]

OR

Subtract the whole numbers and fractions separately.

\[ -4\frac{1}{3} - 4\frac{2}{3} - 4\frac{10}{15} - 4\frac{10}{15} \]

\[ = 3\frac{1}{15} \]

If necessary, borrow a fraction equal to 1 from the whole number.
DECIMALS

Method:

To determine which of two decimals is larger:

1. Write the decimals so that they have the same number of digits (by adding zeros).
2. Start at the left and compare corresponding digits. The larger number will have the larger digit.

Examples

Ex: \(.257 < .31\)
   since \(.257 < .310\)
   and \(2 < 3\)

To round a decimal:

1. Locate the place for which the round off is required.
2. Compare the first digit to the right of this place to 5. If this digit is less than 5, drop it and all digits to the right of it. If this digit is greater than 5, increase the rounded digit by one and drop all digits to the right.

Examples

Ex: Round: \(1.5725\) to the
   a) nearest hundredth
   b) nearest thousandth
   a) \(1.5725 = 1.57\)
   b) \(1.5725 = 1.573\)

To add or subtract decimals:

1. Write the numbers vertically and line up the decimal points. If needed, add zeros to right of decimal digits.
2. Add or subtract as with whole numbers.
3. Align the decimal point in the answer with the other decimal points.

Examples

Ex: Add:
\[
3.65 + 12.2 + .51
\]
\[
3.65
\]
\[
12.20
\]
\[
+.51
\]
\[
16.36
\]

To multiply decimals:

1. Multiply the numbers as whole numbers.
2. Determine the sum of decimal places in the 2 numbers.
3. Make sure the answer has the same number of decimal places as the sum from Step 2. (Insert zeros to the left if necessary.)

Examples

Ex: Multiply:
\[
.0023 \times .14
\]
\[
.0023
\]
\[
\times .14
\]
\[
092
\]
\[
23
\]
\[
.000322
\]
6 decimal places in result
Decimals cont’d

Method:

To divide decimals:
1. Make the divisor a whole number by moving the decimal point to the right. (Mark this position with a caret ^.)
   Ex: Divide: 0.168 by 0.05
   \[
   0.05 \overline{)0.168}
   \]
   Move the decimal in the dividend to the right the same number of places. (Mark this position with a caret ^.)
   \[
   0.168 \\
   168 \\
   3 \bigg( \\
   \]
   Place the decimal point in the answer directly above the caret.
   \[
   3.36 \\
   .05 \overline{)0.1680} \\
   15 \bigg( \\
   18 \\
   15 \\
   30 \\
   30 \\
   0
   \]
   Divide as with whole numbers, adding zeros to the right if necessary. Continue until the remainder is zero, the decimal digits repeat, or the desired number of decimal positions is achieved.

To convert a fraction to a decimal, divide the denominator into the numerator.
Ex: Convert: \( \frac{11}{9} \) to a decimal
\[
11 \overline{)9.0000} = .81
\]

PERCENTS

Technique:

% means "per 100" or "out of 100"

To convert a percentage to a fraction or decimal, divide by 100%.

Note: A shortcut for dividing by 100 is moving the decimal 2 places to the left.

Examples
Ex: 45% means \( \frac{45}{100} \) or 45 out of 100.
Ex: 100% is equal to \( \frac{100}{100} \) or 1
Ex: Convert: 32% to a fraction.
\[
32\% = \frac{32}{100} = \frac{8}{25}
\]
Ex: Convert: 2.5% to a decimal.
\[
2.5\% = \frac{2.5}{100} = .025
\]
To convert a fraction or decimal to a percentage, multiply by 100%

Note: A shortcut for multiplying by 100 is moving the decimal 2 places to the right.

To solve percent equations:
1. Change the percent to a decimal or fraction.
2. Translate the question to an equation by replacing "is" with =, "of" with multiplication, and "what" with a variable.
3. Solve the equation for the variable.

To solve a percent increase or decrease problem, use the following models:

Increase
\[
\text{new} = \text{original} + \text{percent as decimal} \times \text{original}
\]

Decrease
\[
\text{new} = \text{original} - \text{percent as decimal} \times \text{original}
\]

Ex: Convert: \( \frac{3}{5} \) to a percent.
\[
\frac{3}{5} \times 100\% = \frac{3}{5} \times \frac{100\%}{1} = 60\%
\]

Ex: Convert: 1.4 to a percent
\[1.4 \times 100\% = 140\%
\]

Ex: 14 is 25\% of what number?
\[
\frac{14}{.25} = x
\]
\[
56 = x
\]
so 14 is 25\% of 56

Ex: If 68,000 was increased to 78,500, find the percent increase.
\[
\begin{align*}
78,500 &= 68,000 + x(68,000) \\
10,500 &= 68,000x \\
10,500 &= 68,000x \\
68,000 &= 68,000 \\
.154 &= x
\end{align*}
\]
So the percent increase is 15.4\%
Technique:  

To convert units, multiply by a fraction consisting of a quantity divided by the same quantity with different units (multiplication by 1). Set up the fraction so that the units will cancel appropriately.

Examples

Ex: Convert 5 meters to feet using the fact that 1 ft = .305 meters

\[ 1\text{ ft} = .305\text{ meters} \]

\[ 5\text{ meters} \left( \frac{1\text{ ft}}{.305\text{ meters}} \right) = \frac{5\text{ ft}}{.305} = 16.39\text{ ft} \]

Ex: How long does it take a car traveling 55 mph to travel 30 miles?

\[ d = rt \]

\[ 30 = 55t \]

\[ \frac{30}{55} = t \]

\[ t = .55 \text{ hours or 33 minutes} \]

Ex: A plane flew in a straight line to a point 100 miles west and 150 miles north from where it began. How far did that plane travel?

![Diagram](image)

\[ c^2 = 100^2 + 150^2 \]

\[ c^2 = 32500 \]

\[ c = \sqrt{32500} \]

\[ c = 180.28 \text{ miles} \]

The plane flew 180.28 miles
### TRANSLATING PHRASES TO ALGEBRAIC EXPRESSIONS

<table>
<thead>
<tr>
<th>Verbal Description</th>
<th>Algebraic Operation or Symbol</th>
<th>Examples</th>
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</table>
| is, equal, are, results in | equal sign | a number plus 7 results in 10  
\[ x + 7 = 10 \] |
| sum, plus, increased, by, greater than, more than, exceeds, total of | addition | the sum of a number and 2  
\[ x + 2 \] |
| difference, minus, decreased by, less than, subtracted from, reduced by, the remainder | subtraction | 7 subtracted from 5  
\[ 5 - 7 \] |
| product, multiplied by, twice, times, of | multiplication | twice a number  
\[ 2x \] |
| quotient, divided by, ratio, per | division | 35 miles per hour  
\[ \frac{35 \text{ miles}}{1 \text{ hour}} \] |
| exponent, power, squared, cubed | exponent | two cubed  
\[ 2^3 \]  
\[ x^5 \]  
five times the sum of a number and 3  
\[ 5(x + 3) \] |

**Note:** Parentheses must be used to indicate an operation is to be applied to an entire expression.

**Note:** Use a variable to represent an unknown quantity.
## SETS OF REAL NUMBERS

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<th><strong>Definitions</strong></th>
<th><strong>Examples</strong></th>
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<td><strong>Integers</strong> = all positive and negative whole numbers and zero.</td>
<td>Ex: -100, 20, 0, -451 (all integers)</td>
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<td><strong>Rational Numbers</strong> = all terminating or repeating decimals.</td>
<td>Ex: ( \frac{1}{4} = .25 ) and ( \frac{2}{3} = .6 ) are rational</td>
</tr>
<tr>
<td><strong>Irrational Number</strong> = all nonterminating, nonrepeating decimals.</td>
<td>Ex: ( \sqrt{2} = 1.4142135... ) is irrational. Ex: ( \pi = 3.1415926... ) is irrational.</td>
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<tr>
<td><strong>Prime Number</strong> = positive integer greater than 1 with no integer factors other than itself and 1.</td>
<td>Ex: 5, 17, 29, and 37 (all prime)</td>
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### Real Number Line

Ex: Plot \( \frac{5}{3} \) on the number line. \( \frac{5}{3} = 1 \frac{2}{3} \)

### Absolute Value

\( |a| \) = the distance between \( a \) and 0 on the number line.

Ex: 2 = 2
0 = 0
-3 = 3
OPERATIONS ON INTEGERS

**Techniques:**

**Addition**
- **If the numbers have the same signs,** add the absolute value and attach the common sign to the result.
  - Ex: \(5 + 12 = 17\)
  - Ex: \(-4 + (-10) = -14\)
  - Ex: \((-3) + (-7) + (-10) = -20\)

- **If the numbers have opposite signs,** subtract the smaller absolute value from the larger and attach the sign of the larger.
  - Ex: \(5 + (-12) = -7\)
  - Ex: \((-7) + 3 = -4\)
  - Ex: \(-2 + 10 = 8\)

**Subtraction**
- Add the opposite of the second number.
- Subtracting a positive is the same as adding a negative.
- Subtracting a negative is the same as adding a positive.
  - Ex: \(7 - 15 = 7 + (-15) = -8\)
  - Ex: \(4 - 5 = 4 + (-5) = -1\)
  - Ex: \(7 - (-3) = 7 + 3 = 10\)

**Multiplication and Division**
- **If the numbers have the same signs,** perform the operation on the absolute values and attach a positive sign to the result.
  - Ex: \((-6) \div (-2) = 3\)
  - Ex: \(-12 \div -4 = 3\)
  - Ex: \((-3)(-2) = 6\)
  - Ex: \((-5)(2) = -10\)
  - Ex: \(-15 \div 3 = -5\)

- **If the numbers have opposite signs,** perform the operation on the absolute values and attach a negative sign to the result.

**Exponents (positive integer exponents)**
- Multiply the base the number of times given by the exponent.
  - Ex: \((-2)^3 = (-2)(-2)(-2) = -8\)
  - Ex: \(2^5 = (2)(2)(2)(2)(2) = 32\)
  - Ex: \(-2^4 = -1(2)(2)(2)(2) = -16\)
  - Ex: \((-2)^4 = (-2)(-2)(-2)(-2) = 16\)
ORDER OF OPERATIONS

Techniques:

To evaluate an expression using the order of operations, perform operations in the following order:

1. **Parentheses**
   Starting with the innermost symbol, perform operations inside symbols of grouping (parentheses or brackets) or absolute value symbols.

2. **Exponents**
   Evaluate all exponential expressions.

3. **Multiplication/Division**
   In order from left to right, perform all multiplications and divisions.

4. **Addition/Subtraction**
   In order from left to right, perform all additions and subtractions.

Examples

Ex:  \[ \text{Evaluate } 16 \div 2^3 - 4(3 - |5 - 7|) + 5 \]

\[= 16 \div 2^3 - 4(3 - |5 - 7|) + 5\]

\[= 16 \div 2^3 - 4(3 - |5 - 7|) + 5\]

\[= 16 \div 8 - 4(1) + 5\]

\[= 2 - 4 + 5\]

\[= 3\]
EVALUATING ALGEBRAIC EXPRESSIONS

Techniques:

To evaluate an expression at given values of the variables:

1. Replace every occurrence of each variable with the appropriate real number. Use parentheses when substituting a negative number for a variable.

2. Use the order of operations to evaluate the resulting expression.

Examples

Ex: Evaluate \( y^2 - x + 2z + z^3 \)
when:
\( x = -3, \ y = -5 \) and \( z = 2 \)

\[
\begin{align*}
\ y^2 \ - \ x \ + \ 2z \ + \ z^3 \\
= (-5)^2 \ - \ (-3) \ + \ 2(2) \ + \ 2^3 \\
= 25 \ + \ 3 \ + \ 4 \ + \ 8 \\
= 40
\end{align*}
\]

SIMPLIFYING ALGEBRAIC EXPRESSIONS

Techniques:

To simplify an algebraic expression:

1. Starting with the innermost set, remove symbols of grouping. Usually the distributive property and/or rules of exponents must be used in this step.

2. Combine like terms by adding the coefficients of terms having the same variable factor.

Examples

Ex: Simplify:
\( (5x)^2 + 4 \ [x^2 - (2x - 5)] \)

\[
\begin{align*}
= (5x)^2 + 4 \ [x^2 - 2x + 5] \\
= 25x^2 + 4x^2 - 8x + 20 \\
= 29x^2 - 8x + 20
\end{align*}
\]
PERIMETER, AREA, AND VOLUME

Formulas:     Examples

The following notation is used in the formulas:
l = length, w = width, h = height,
b = base, r = radius, a = area, v = volume
c = circumference, p = perimeter

Square        a = w²

\[\begin{array}{c}
5 \text{ in} \\
5 \text{ in}
\end{array}\]

5 in.       a = 25in²

Rectangle     a = lw

\[\begin{array}{c}
6 \text{ cm} \\
7 \text{ cm}
\end{array}\]

6 cm       a = 42cm²

Parallelogram a = bh

\[\begin{array}{c}
6 \text{ ft} \\
5 \text{ ft} \\
10 \text{ ft}
\end{array}\]

a = 10(5) = 50ft²

Triangle      a = ½ bh

\[\begin{array}{c}
7 \text{ in} \\
6 \text{ in} \\
12 \text{ in} \\
10 \text{ in}
\end{array}\]

a = \(\frac{1}{2} (12)(6) = 36\text{in²}\)

Circle        a = \(\pi r²\)  \(\pi = 3.14\)

\[\begin{array}{c}
4 \text{ in}
\end{array}\]

a = \(\pi(4)^2 = 50.27\text{in²}\)

\(c = 2\pi r\)

\[\begin{array}{c}
4 \text{ in}
\end{array}\]

c = 2 \(\pi(4) = 25.13\text{in}\)

Any Polygon   P= sum of the sides

\[\begin{array}{c}
6 \text{ in} \\
6 \text{ in} \\
10 \text{ in} \\
13 \text{ in}
\end{array}\]

P = 6 + 6 + 13 + 10 = 35in
## Perimeter, Area, and Volume cont’d

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<td>$V = w^3$</td>
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<td>$V = 125\text{cm}^3$</td>
<td>![Cube Diagram]</td>
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<td>5 cm</td>
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<th>$v = lwh$</th>
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<td>$V = 9 \times 4 \times 5$</td>
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<td>9 in</td>
<td>$V = 180\text{in}^3$</td>
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<th>$v = \pi r^2 h$</th>
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<td>2 in radius</td>
<td>$V = \pi 2^2 (4)$</td>
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<tr>
<td>4 in height</td>
<td>$V \approx 50.27\text{in}^3$</td>
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<th><strong>Sphere</strong></th>
<th>$v = \frac{4}{3} \pi r^3$</th>
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<tr>
<td>3 in</td>
<td>$V = \frac{4}{3} \pi (3)^3$</td>
</tr>
<tr>
<td></td>
<td>$V \approx 113.1\text{in}^3$</td>
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LINEAR EQUATIONS

Techniques:

To solve a linear equation:

1. Remove fractions by multiplying both sides by the least common denominator (LCD).
   Ex: Solve \( \frac{2}{3}(x + 2) + \frac{5}{6}x = x + 4 \)
   LCD = 6 so multiply each term by 6 on both sides
   \[ 6 \cdot \frac{2}{3}(x + 2) + 6 \cdot \frac{5}{6}x = 6(x + 4) \]

2. Simplify each side by distributing and combining like terms.
   \[ 4(x + 2) + 5x = 6(x + 4) \]
   \[ 4x + 8 + 5x = 6x + 24 \]

3. Add or subtract variable terms from both sides so that the variable will occur on only one side.
   \[ 9x + 8 = 6x + 24 \]
   \[ -6x -6x \]
   \[ 3x + 8 = 0 + 24 \]

4. Isolate the variable by performing the same operation on both sides of the equation.
   \[ 3x + 8 = 24 \]
   \[ -8 -8 \]
   \[ 3x = 16 \]
   \[ \frac{3x}{3} = \frac{16}{3} \]
   \[ x = \frac{16}{3} \]

Note: A check is necessary if the equation contains fractions with variable denominators.

To solve an application problem using a linear equation.

Ex: A person has 90 coins in quarters and dimes with a combined value of $16.80. Determine the number of coins of each type.

Value of quarters + value of dimes = total value

1. Write a verbal equivalence containing the quantities involved in the problem.

2. Substitute given values for known quantities.
   \[ .25(number \ of \ quarters) + .10(number \ of \ dimes) = total \ value \]
   \[ .25(number \ of \ quarters) + .10(number \ of \ dimes) = 16.80 \]

3. Use a variable to represent one unknown quantity in the equation.
   \[ .25x + .10(number \ of \ dimes) = 16.80 \]

4. Replace the remaining unknown quantities with an appropriate expression involving the variable.
   \[ .25x + .10(90 - x) = 16.80 \]
   \[ .25x + 9 - .10x = 16.80 \]
   \[ .15x + 9 = 16.80 \]
   \[ 15x = 7.8 \]
   \[ x = 52 \]

5. Solve the equation.
   \[ x = 52 \]
   \[ 90 - x = 90 - 52 = 38 \]
   There are 52 quarters and 38 dimes.
INEQUALITIES

**Techniques:**

To solve a *linear inequality*, employ the same procedure used for solving equations, but remember to reverse the order symbol when multiplying or dividing both sides by a negative number.

To solve a *compound inequality*, isolate the variable in the center by performing the same operation on all three parts.

**Examples**

Ex: Solve $-2x < 20$

\[ -2x < 20 \]
\[ -2 \quad -2 \text{ (divided by negative)} \]
\[ x > -10 \text{ (reverse direction of symbol)} \]

Ex: Solve $-13 \leq 6x - 1 < 3$

\[ -13 \leq 6x - 1 < 3 \]
\[ +1 \quad +1 \quad +1 \]
\[ -12 \leq 6x < 4 \]
\[ 6 \quad 6 \quad 6 \]
\[ \frac{-12}{6} \leq x < \frac{4}{6} \]
\[ -2 \leq x < \frac{2}{3} \]

Ex: Graph $x + 2 < 1$

\[ x + 2 < 1 \]
\[ -2 \quad -2 \]
\[ x < -1 \]

Ex: Write $-6 \leq x < 3$ in interval notation

\[ [-6, 3) \]

Ex: Write $x < 2$ in interval notation

\[ (-\infty, 2) \]

(-\infty \text{ is used since there is no smallest number that is less than 2.})
EXPO**NENTS**

**Rules:**

A **positive integer exponent** dictates the number of times the base is to be multiplied.

A **negative integer exponent** applied to a base is equal to the reciprocal of the base raised to the opposite exponent.

The **zero exponent** applied to any base (except 0) is equal to 1.

A factor can be moved from numerator to denominator (or vice versa) by changing the sign of the exponent.

An exponent can be applied to each part of a **product** or **fraction**.

To apply an exponent to a sum or difference, multiply the polynomial by itself.

**Examples**

Ex: \( 3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81 \)

Ex: \((-1)^5 = (-1)(-1)(-1)(-1)(-1) = 1 \cdot 1 \cdot (-1) = -1 \)

Ex: \( 2^{-3} = \frac{1}{2^3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8} \)

Ex: \( x^{-3} = \frac{1}{x^3} \)

Ex: \( 4^0 = 1 \)

Ex: \((-12)^0 = 1 \)

Ex: \(- (12)^0 = -1 \)

Ex: \( \frac{3 \cdot 4^2}{2^3} = \frac{3 \cdot 2^3}{4^2} = \frac{9 \cdot 8}{16} = \frac{9}{2} \)

Ex: \( \frac{3xy^3}{z^3} = \frac{3xz^4}{y^3} \)

Ex: \( \left( \frac{2}{3} \right)^3 = \frac{2^3}{3^3} = \frac{8}{27} \)

Ex: \((3xy)^2 = 3^2 x^2 y^2 = 9x^2 y^2 \)

Ex: \((x + 1)^2 = (x + 1)(x + 1) = x^2 + 2x + 1 \)
OPERATIONS ON EXPONENTS

Procedures:

To multiply expressions with the same base, keep the base and add the exponents.

Examples

Ex: \( x^2 \cdot x \cdot x^4 = x^{2+1+4} = x^7 \)

To divide expressions with the same base, keep the base and subtract the exponents.

Examples

Ex: \( \frac{y^9}{y^5} = y^{9-5} = y^4 \)

Ex: \( \frac{x^3}{x^7} = x^{3-7} = x^{-4} = \frac{1}{x^4} \)

To raise an exponential expression to another power, keep the base and multiply the exponents.

Examples

Ex: \( (x^3)^4 = x^{3(4)} = x^{12} \)

To add or subtract expressions with exponents, combine like terms. The base and the exponent must be the same.

Examples

Ex: \( 3x^2 + 5x^2 = 8x^2 \)

Ex: \( 4x^2 - 3x - 6x^2 = 4x^2 - 6x^2 - 3x = -2x^2 - 3x \)

OPERATIONS ON POLYNOMIALS

Techniques:

To add polynomials, combine like terms (terms with the same variable raised to the same exponent.)

Examples

Ex: Simplify:
\[
\begin{align*}
(4x^3 - 2x^2 - 7x) + (-6x^3 - 3x^2 + 5) &= 4x^3 - 2x^2 - 7x - 6x^3 - 3x^2 + 5 \\
&= 4x^3 - 6x^3 - 2x^2 - 3x^2 - 7x + 5 \\
&= -2x^3 - 5x^2 - 7x + 5
\end{align*}
\]

To subtract polynomials:

1. Distribute the \(-1\). (This will change the sign of each term on the 2\textsuperscript{nd} polynomial.)

2. Combine like terms.

Examples

Ex: Simplify:
\[
\begin{align*}
(4x^3 - 2x^2 - 7x) - (-6x^3 - 3x^2 + 5) &= 4x^3 - 2x^2 - 7x + 6x^3 + 3x^2 - 5 \\
&= 4x^3 + 6x^3 - 2x^2 + 3x^2 - 7x - 5 \\
&= 10x^3 + x^2 - 7x - 5
\end{align*}
\]
To multiply polynomials:
1. Multiply each term of one polynomial by each term of the other polynomial. (This is actually the distributive property applied more than once.)

2. Combine like terms.

**Note:** In the case of multiplying 2 binomials, the method is often referred to as FOIL for First, Outer, Inner, Last.

To divide a polynomial by a monomial, divide the monomial into each term of the polynomial.

To divide a polynomial, use the long division pattern for dividing whole numbers:

1. Arrange both polynomials in standard form with exponents in descending order. If either polynomial has a “missing term,” use zero as a placeholder.

2. Divide the first term of expression by 1st term of divisor.

3. Multiply the result by each term.

4. Subtract by changing each sign and combining like terms.

5. Bring down the next term in the dividend.

6. Repeat steps 1-4 until the process is complete.

7. Add to the resulting quotient the remainder divided by the divisor.

---

**Ex:** Multiply \((2x^2 - 7x + 1)(3x + 4)\)

\[
(2x^2 - 7x + 1)(3x + 4) = (2x^2)(3x) + 2x^2(4) - 7x(3x) - 7x(4) + 1(3x) + 1(4)
\]

\[
= 6x^3 + 8x^2 - 21x^2 - 28x + 3x + 4
\]

\[
= 6x^3 - 13x^2 - 25x + 4
\]

**Ex:** Multiply \((x + 3)(x - 2)\)

\[
(x + 3)(x - 2) = x^2 - 2x + 3x - 6
\]

F O I L

\[
x^2 + x - 6
\]

**Ex:** Divide \(3x^2 + 6x + 10\)

\[
\frac{3x^2 + 6x + 10}{2x} = \frac{3x^2}{2x} + \frac{6x}{2x} + \frac{10}{2x}
\]

\[
= \frac{3x}{2} + 3 + \frac{5}{x}
\]

**Ex:** Divide \(\frac{x^2 - 3}{x + 2}\)

\[
\frac{x - 2}{x + 2} + \frac{1}{x + 2}
\]

\[
\frac{-x^2 + 2x}{-2x - 3} + \frac{2x}{2x} + 4
\]

\[
\frac{1}{1}
\]
FACTORING

Techniques:

To factor a polynomial:

1. Factor using the greatest common factor (GCF). (Divide each term by the largest expression that will divide into every term.)

   Ex: Factor $8x^3 - 50x$ (2 terms)
   
   $8x^3 - 50x = 2x(4x^2 - 25)$
   
   = $2x(2x + 5)(2x - 5)$
   
   difference of squares

2. Use the factoring technique that corresponds to the number of terms.

   (a) If there are 2 terms, use the difference of squares or the difference or sum of cubes formula.
   
   $a^2 - b^2 = (a + b)(a - b)$
   
   $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
   
   $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

   (b) If there are 3 terms, use the reverse of FOIL trial-and-error technique.

   Ex: Factor $3x^3 - 24$ (2 terms)
   
   $3x^3 - 24 = 3(x^3 - 8)$
   
   = $3(x - 2)(x^2 + 2x + 4)$
   
   difference of cubes

   (c) If there are 4 terms, use the grouping technique. (Factor the GCF out of the first 2 terms, then out of the second 2 terms, then out of the resulting final expression.

   Ex: Factor $x^3 - 2x^2 + 4x - 8$ (4 terms)
   
   $x^3 - 2x^2 + 4x - 8$ Grouping
   
   = $x(x - 2) + 4(x - 2)$
   
   = $(x - 2)(x^2 + 4)$

   Check
   
   $(x - 2)(x^2 + 4)$
   
   = $x^3 + 4x - 2x^2 - 8$ ✓
   
   F O I L

3. Check each step of the factoring with multiplication
RATIONAL EXPRESSIONS

**Techniques:**

**To simplify a rational expression,** completely factor the numerator and denominator and then cancel common factors.

Ex: Simplify:

\[
\frac{x^2 + 2x - 15}{3x - 9}
\]

\[
x^2 + 2x - 15 = (x + 5)(x - 3) = \frac{x + 5}{3(x - 3)} = \frac{1}{3}
\]

**To multiply or divide rational expressions:**

1. Completely factor the numerator and denominator.
2. Perform the indicated operation.
3. Cancel common factors.

Ex: Divide:

\[
\frac{2x}{3x - 12} \div \frac{x^2 - 2x}{3x - 12}
\]

\[
\frac{2x}{3(x - 4)} \times \frac{(x - 4)(x - 2)}{x(x - 2)} = \frac{2}{3}
\]

Ex: Multiply:

\[
\frac{x}{5x^2 - 20x} \times \frac{x - 4}{2x^2 + x - 3}
\]

\[
\frac{x}{5(x - 4)} \times \frac{(2x + 3)(x - 1)}{1} = \frac{5(2x + 3)(x - 1)}{(x - 2)^2}
\]

**To add or subtract rational expressions:**

1. Completely factor the numerator and denominator.
2. Find the least common denominator by using each factor represented, raised to the highest power occurring in each denominator.
3. Multiply numerator and denominator by an expression resulting in the common denominator.
4. Perform the operation and simplify.

Ex: Subtract:

\[
\frac{6x}{x^2 - 4} - \frac{3}{(x - 2)^2}
\]

\[
= \frac{6x}{(x + 2)(x - 2)} - \frac{3}{(x - 2)^2} \quad \text{LCD} = (x + 2)(x - 2)^2
\]

\[
= \frac{6x(x - 2)}{(x + 2)(x - 2)^2} - \frac{3(x + 2)}{(x + 2)(x - 2)^2}
\]

\[
= \frac{6x^2 - 12x}{(x + 2)(x - 2)^2} - \frac{3x + 6}{(x + 2)(x - 2)^2}
\]

\[
= \frac{6x^2 - 15x - 6}{(x + 2)(x - 2)^2}
\]
Points:
A point on the rectangular coordinate system can be represented by an ordered pair (x,y). The first coordinate gives the position along the horizontal axis, and the second gives the vertical position.

Examples
Ex: (0, -2) and (-3, 2)

Intercepts:
To find the x-intercept of the graph of a given equation, let y = 0 and solve for x.
Ex: Find the x and y of the graph of 3x + y = 6
x-intercept: y-intercept:
3x + 0 = 6
3x = 6
x = 2
(2, 0)
3(0) + y = 6
y = 6
(0, 6)

To find the y-intercept of the graph of a given equation, let x = 0 and solve for y.
Ex: Sketch: the graph of

Point Plotting:
The graph of an equation is the plotted set of all ordered pairs whose coordinators satisfy the equation.

To sketch a graph using the point-plotting method:
1. Isolate one of the variables.

2. Make a table of values showing several solution points.

3. Plot the points in a rectangular coordinating system.

4. Connect these points with a smooth curve or line.
**Lines:**

To find the slope of the line through 2 points, use the following formula:
Given points \((x_1, y_1)\) and \((x_2, y_2)\)
slope \(m = \frac{y_2 - y_1}{x_2 - x_1}\)

To find the equation of the line with given characteristics, use the following formula:
\(y - y_1 = m(x - x_1)\) is the line through the point \((x_1, y_1)\) with slope \(m\)

The graph of \(y = mx + b\) is a line with slope \(m\) and y-int = \((0, b)\).

\(x = a\) is a vertical line through \((a, 0)\) with undefined slope.

\(y = b\) is a horizontal line through \((0, b)\) with slope zero.

**Parallel lines** have the same slopes.

**Perpendicular lines** have slopes that are negative reciprocals.

**Example:** Find the equation of the line through \((-3, 7)\) and \((3, 1)\)

\(m = \frac{7 - 1}{-3 - 3} = \frac{6}{-6} = -1\)

\(y - y_1 = m(x - x_1)\)

\(y - 7 = -1(x + 3)\)

\(y - 7 = -x - 3\)

\(y = -x + 4\)

**Example:** \(y = -x + 4\) is the equation of the line with slope = -1 and y-int = \((0, 4)\)

**Example:** \(x = 2\), slope is undefined

\(y = 3\) slope 0

**Example:** \(y = 2x + 5\) and \(y = \frac{1}{2}x + 3\) are the equations of perpendicular lines since the slopes are 2 and \(-\frac{1}{2}\).
**Parabolas:**
The graph of \( f(x) = ax^2 + bx + c \) where
\( a \neq 0 \) is a parabola with vertex \( \left( \frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \).

**If a is positive,** the parabola opens up.

**If a is negative,** the parabola opens downward.

**Ex:**
The graph of \( f(x) = x^2 + 2x + 4 \) is a parabola which opens upward that has a vertex \((-1, 3)\).

\[
x = \frac{-b}{2a} = \frac{-2}{2} = -1
\]

\[
y = f(-1) = (-1)^2 + 2(-1) + 4
= 1 - 2 + 4 = 3
\]

---

**SIMPLIFYING RADICALS**

**Techniques:**

**To remove all possible factors from the radical:**

1. Write the number as a product using the largest factor that is a perfect \( k \)th power where \( k \) is index.

2. If possible, write the variable factor as a product using the largest exponent that is a multiple of \( k \).

3. Apply the radical to each part of the fraction or product. The roots are written outside radical and the "leftover" factors remain under the radical.

**Examples**

**Ex:** Simplify \( \sqrt{50} \)

\[
\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \sqrt{2} = 5\sqrt{2}
\]

**Ex:** Simplify \( \sqrt[3]{54x^6y^8} \)

\[
\sqrt[3]{54x^6y^8} = \sqrt[3]{27 \cdot 2x^6y^6y^2}
= 3x^2y^2 \sqrt[3]{2y^2}
\]
Simplifying Radicals cont’d

To rationalize a denominator with one term,

multiply the numerator and denominator by a radical that will produce a perfect kth power radicand in the denominator and simplify.

**Ex:** Rationalize the denominator in \( \frac{1}{\sqrt[3]{x^2}} \)

\[
\frac{1}{\sqrt[3]{x^2}} = \frac{1}{x} \cdot \frac{\sqrt[3]{x}}{\sqrt[3]{x}} = \frac{\sqrt[3]{x}}{x}
\]

To rationalize a denominator with two terms, multiply the numerator and denominator by the conjugate of denominator (opposite middle sign), then multiply using FOIL and simplify.

**Ex:** Rationalize the denominator in \( \frac{2}{3 - \sqrt{5}} \)

\[
\frac{2}{3 - \sqrt{5}} = \frac{2}{(3 - \sqrt{5})(3 + \sqrt{5})} = \frac{2(3 + \sqrt{5})}{9 - 5} = \frac{2(3 + \sqrt{5})}{4}
\]

**Techniques:**

To reduce the index of a radical,

rewrite using a fractional exponent and reduce the fraction before converting back to radical notation.

**Note:** If the fraction can not be reduced, try writing the radican using an exponent.

**Note:** \( \sqrt{-1} = i \), an imaginary number.

**Examples**

**Ex:** Simplify: \( 6\sqrt{x^2} \)

\[
6\sqrt{x^2} = x^{\frac{2}{6}} = x^{\frac{1}{3}} = \sqrt[3]{x}
\]

**Ex:** Simplify: \( \sqrt[4]{9} \)

\[
\sqrt[4]{9} = 9^{\frac{1}{4}} = (3^2)^{\frac{1}{4}} = 3^{\frac{2}{4}} = 3^{\frac{1}{2}} = \sqrt{3}
\]

**Ex:** Simplify \( \sqrt{-8} \)

\[
\sqrt{-8} = \sqrt{-4 \cdot 2} = \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{2} = i 2\sqrt{2} = 2i\sqrt{2}
\]
OPERATIONS ON RADICALS

Techniques:

A **fractional exponent** indicates that a **radical** should be applied to the base. The **numerator** of the exponent denotes the **power** to which the base is raised, and the **denominator** denotes the **root** to be taken.

### Examples

Ex: $8^{\frac{3}{4}} = \sqrt[4]{8^3} = \sqrt[4]{64} = 4$

or

$8^{\frac{3}{2}} = (\sqrt[2]{8})^3 = 2^3 = 4$

---

To add and subtract radicals (combine like radicals): Ex:

Add: $\sqrt{75} + \sqrt{27}$

1. Simplify each radical.  
   
   $\sqrt{75} + \sqrt{27} = \sqrt{25 \cdot 3} + \sqrt{9 \cdot 3}$

2. Combine those having same index and radicand by adding/subtracting their coefficients.  
   
   $= 5\sqrt{3} + 3\sqrt{3} = 8\sqrt{3}$

---

To multiply radicals with the same indices, multiply the radicands.  

Ex: $\sqrt{5} \cdot \sqrt{2} = \sqrt{5(2)} = \sqrt{10}$

---

To divide radicals with the same indices, divide the radicands.  

Ex: $\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{\frac{27}{3}} = \sqrt{9} = 3$

---

To multiply and divide radicals with different indices:

Ex: Multiply $\sqrt{x} \cdot \frac{1}{\sqrt{x}}$

1. Write each radical with fractional exponents.

   $\sqrt{x} \cdot \sqrt{x} = x^{\frac{1}{2}} \cdot x^{\frac{1}{2}}$

   $= x^{\frac{3}{2}} x^\frac{3}{6}$

   $= x^\frac{6}{6} = \sqrt{x^6}$

OR

2. Rewrite each with a common denominator.

3. Convert to the radical form.

   $\sqrt{x} \cdot \sqrt{x} = x^{\frac{1}{2}} x^{\frac{1}{2}}$

   $= x^{\frac{3}{6}} \cdot x^\frac{3}{6}$

   $= x^\frac{6}{6} \cdot x^\frac{3}{6}$

4. Multiply or divide as usual.

   $= x^\frac{6}{6} \cdot x^\frac{3}{6}$

   $= x^\frac{6}{3} \cdot x^\frac{3}{3}$

   $= x^\frac{6}{3} \cdot x^\frac{3}{3}$

   $= x^\frac{9}{3} \cdot x^\frac{3}{3}$

   $= x^\frac{9}{3}$

---

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SPECIAL EQUATIONS

Techniques:

**Quadratic Equation** – (2<sup>nd</sup> degree equation) -
equation that can be written in the form \( ax^2 + bx + c = 0 \)
where \( a, b, \) and \( c \) are real and \( a \neq 0 \).

To solve:

1. If the equation has the form
   \( ax^2 + c = \) (no x term) isolate
   the squared quantity and extract
   square roots. *Don’t forget the ±.*
   
   **Ex:** Solve \( x^2 - 5 = 0 \)
   \( x^2 - 5 = 0 \) has no x-term
   \( x^2 = 5 \)
   \( x = ±\sqrt{5} \)

2. If zero is isolated and the expression
   \( ax^2 + bx + c \) will factor, then factor,
   set each factor equal to zero, and solve
   each equation.
   
   **Ex:** Solve \( 2x^2 + 9x = 5 \)
   \( 2x^2 + 9x - 5 = 0 \)
   \( (2x - 1)(x + 5) = 0 \)
   \( 2x - 1 = 0 \) \( x + 5 = 0 \)
   \( x = \frac{1}{2} \) \( x = -5 \)

3. The Quadratic Formula can be
   used on any quadratic equation.
   Set one side equal to zero, identify
   \( a, b, \) and \( c \) and substitute into formula.
   
   **Ex:** Solve \( x^2 + 7x + 4 = 0 \)
   \( x^2 + 7x + 4 = 0 \) will not factor
   \( a=1, b=7, c=4 \)
   \( x = \frac{-7 ± \sqrt{49 - 4(1)(4)}}{2(1)} \)
   \( x = \frac{-7 ± \sqrt{33}}{2} \)
Special Equations cont’d

Techniques:  

**Radical Equation** – equation in which the variable occurs in a radical or is raised to a fractional exponent.

To solve:
1. Isolate the most complicated radical on one side.
2. Raise each side to the power equal to the index of the radical.
3. If the radical remains, repeat steps 1 and 2.
4. Solve the resulting equation.
5. A check is necessary if the original equation involves a radical with an even index.

**Examples**

**Ex:** Solve $\sqrt{x + 2} + 4 = 10$

\[ \begin{align*} 
\sqrt{x + 2} + 4 &= 10 \\
\sqrt{x + 2} &= 6 \\
(\sqrt{x + 2})^2 &= 6^2 \\
x + 2 &= 36 \\
x &= 34 
\end{align*} \]

Check:

\[ \begin{align*} 
\sqrt{34 + 2} + 4 &= 10 \\
\sqrt{36 + 4} &= 10 \\
6 + 4 &= 10 \\
10 &= 10 √ 
\end{align*} \]

**Higher - Order Factorable Equation** - equation in which zero is isolated and the polynomial on the other side is factorable.

To solve:
1. Isolate zero
2. Factor.
3. Set each factor equal to zero and solve each equation.

**Examples**

**Ex:** Solve $x^3 = 4x$

\[ \begin{align*} 
x^3 &= 4x \\
x^3 - 4x &= 0 
\end{align*} \]

\[ \begin{align*} 
&= x(x^2 - 4) \\
&= x(x + 2)(x - 2) = 0 
\end{align*} \]

\[ \begin{align*} 
x &= 0 \\
x + 2 &= 0 \\
x - 2 &= 0 \\
x &= 0 \text{ or } x = -2 \text{ or } x = 2 
\end{align*} \]
MORE SAMPLE QUESTIONS:
More sample questions, including math questions, are available at the following link:


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