NC DAP
(NORTH CAROLINA DIAGNOSTIC ASSESSMENT AND PLACEMENT)

MATHEMATICS STUDY GUIDE
(REvised 11.29.18)
All students are encouraged to prepare for placement testing. Not reviewing for the placement tests could result in students being placed into courses below their actual skill level. This can delay student progress, so prepare as best you can!

The placement test will determine which English and math courses you will take when you attend Coastal. All of our courses are designed to help you succeed, so you will be in good hands, no matter where you place.

This study guide contains reminders, testing tips, an overview of the test, and sample questions.

The faculty and staff of Coastal Carolina Community College wish you the best of luck as you embark on your educational path. We look forward to working with you!

**Reminders**
- The test is computerized; you will be furnished scrap paper and pencil to make notes and/or calculations.
- Bring a photo ID and your student ID number on test day.
- With the exception of the essay (2 hour limit), each section is untimed.
- Once an answer is submitted, it cannot be changed.
- Unauthorized devices such as cell phones and tablets are not allowed.
- Work by yourself.

**Testing Tips**
- Get plenty of rest the night before you plan to take the test.
- Make sure you eat a good breakfast or lunch prior to testing.
- Take the test seriously; you may only test twice in a 12 month period.
- Don’t be discouraged; this test is designed to feel difficult.
- Write as much as you can for this essay; don’t just stop when you reach the required word count. whatever you do, don’t skip the essay section!
Mathematics Overview
The NCCCS Diagnostic and Placement Mathematics test contains 72 questions that measure proficiency in six content areas. *This test is untimed; you may stop the test and resume at a later time.* The six content areas are as follows:

**DMA 010: Operations with Integers** — Topics covered in this category include:
- Problem events that require the use of integers and integer operations
- Basic exponents, square roots, and order of operations
- Perimeter and area of rectangles and triangles
- Angle facts and the Pythagorean Theorem

**DMA 020: Fractions and Decimals** — Topics covered in this category include:
- Relationships between fractions and decimals
- Problem events that result in the use of fractions and decimals to find a solution
- Operations with fractions and decimals
- Circumference and area of circles
- The concept of $\pi$
- Application problems involving decimals

**DMA 030: Proportions, Ratios, Rates and Percentages** — Topics covered in this category include:
- Conceptual application problems containing ratios, rates, proportions, and percentages
- Applications using U.S. customary and metric units of measurement
- Geometry of similar triangles

**DMA 040: Expressions, Linear Equations and Linear Inequalities** — Topics covered in this category include:
- Graphical and algebraic representations of linear expressions, equations, and inequalities
- Application problems using linear equations and inequalities

**DMA 050: Graphs and Equations of Lines** — Topics covered in this category include:
- Graphical and algebraic representations of lines
- Interpretation of basic graphs (line, bar, circle, etc.)

**DMA 060: Polynomials and Quadratic Applications** — Topics in this category include:
- Graphical and algebraic representations of quadratics
- Finding algebraic solutions to contextual quadratic applications
- Polynomial operations
- Factoring polynomials
- Applying factoring to solve polynomial equations

### DEVELOPMENTAL MATH (DMA) CLASSES

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<th>DEVELOPMENTAL MATH (DMA) CLASSES</th>
<th>STUDY/FOCUS AREAS</th>
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<td>DMA 010</td>
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<td>DMA 070</td>
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<td>DMA 080</td>
<td>Pages 39-43</td>
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</tbody>
</table>
OPERATIONS WITH INTEGERS

For each of the questions below, choose the best answer from the four choices given. You may use the paper you received as scratch paper.

1. On a summer day in Benton, the low temperature of 75ºF was reached at 6 in the morning. The high temperature was reached 9 hours later, after the temperature rose 16ºF. What was the high temperature in Benton that day?

A. 81ºF  
B. 84ºF  
C. 91ºF  
D. 96ºF

2. Which of the four labeled points on the number line above has the greatest absolute value?

A. A  
B. B  
C. C  
D. D

3. \((-2 - 4) \times 8 =\)

A. -48  
B. -16  
C. 16  
D. 48

4. The sum of Cheryl's scores on the first four quizzes in her history class was 364 points. If she scores 96 points on her next quiz, what will be her average score for the five quizzes?

A. 89 points  
B. 91 points  
C. 92 points  
D. 94 points

5. \(\sqrt{529} =\)

A. 17  
B. 23  
C. 26  
D. 27

FRACTIONS AND DECIMALS

For each of the questions below, choose the best answer from the four choices given. You may use the paper you received as scratch paper.

1. A large dining room table is in the shape of a semicircle of diameter 12 feet, as shown above. Of the following, which is closest to the area of the table? (Use \(\pi = 3.14\).)

A. 38 square feet  
B. 57 square feet  
C. 75 square feet  
D. 113 square feet

2. The large square above has area 9 and is divided into 9 smaller squares of equal area. What is the length of the path drawn in bold?

A. 3  
B. 4  
C. 5  
D. 6

3. \(0.6 \div 10^2 =\)

A. 60  
B. 6  
C. 0.06  
D. 0.006

4. \(3,590 =\)

A. \(3.59 \times 10^5\)  
B. \(3.59 \times 10^4\)  
C. \(3.59 \times 10^3\)  
D. \(3.59 \times 10^2\)
PROPORTIONS, RATIOS, RATES, AND PERCENTAGES

For each of the questions below, choose the best answer from the four choices given. You may use the paper you received as scratch paper.

1. During a basketball practice, two players, Sidell and Jeron, each attempted 25 free throws. Sidell made 40% of his free-throw attempts, whereas Jeron made 52% of them. How many more free throws did Jeron make than Sidell?

A. 3  
B. 4  
C. 5  
D. 6

2. A boy skis 4 miles down a mountain slope in 10 minutes. What is his average speed, in miles per hour (mph), over that time interval?

A. 48 mph  
B. 36 mph  
C. 32 mph  
D. 24 mph

3. There are 23 children in a line to buy a hot dog. If every 4th child in line, starting with the fourth in line, gets a toy, what is the ratio of the number of children in line who get a toy to the number of children in line who do not get a toy?

A. 3 : 8  
B. 5 : 23  
C. 5 : 18  
D. 6 : 23

4. 52 is what percent of 160?

A. 30%  
B. 32.5%  
C. 35%  
D. 38.5%

5. Jenna is driving at a speed of 65 miles per hour. What is Jenna's driving speed in kilometers per hour? (There are about 1.6 kilometers in 1 mile.)

A. 112 kilometers per hour  
B. 104 kilometers per hour  
C. 96 kilometers per hour  
D. 92 kilometers per hour

EXPRESSIONS, LINEAR EQUATIONS, AND LINEAR INEQUALITIES

For each of the questions below, choose the best answer from the four choices given. You may use the paper you received as scratch paper.

1. The tick marks on the number line above are equally spaced. The expression $y^2 + 2x$ is equal to

A. $\frac{8}{3}$  
B. 0  
C. $\frac{8}{3}$  
D. $\frac{16}{3}$

2. A party supply store charges an initial charge of $20 to rent a costume plus an additional $8 per day for each day the costume is rented. Which of the following represents the cost, in dollars, to rent a costume for $n$ days?

A. $8n$  
B. $20 + 8n$  
C. $(20)(8n)$  
D. $20-8n$

3. Julie purchased a treadmill that originally cost $t$ dollars at a discount of 8%. Which of the following represents the amount, in dollars, that Julie paid for the treadmill after the discount?

A. $t - 0.8t$  
B. $t + 0.08$  
C. $t + 0.08t$  
D. $t - 0.08t$

4. If $\frac{x}{3} - 2 = 5x - 2$, then $x =$

A. $-\frac{3}{5}$  
B. 0  
C. $\frac{5}{3}$  
D. 1
**GRAPHS AND EQUATIONS OF LINES**

*For each of the questions below, choose the best answer from the four choices given. You may use the paper you received as scratch paper.*

1. The linear equation graphed above gives the amount of money Company H has saved $y$ years after the company opened. According to the graph, how many years after the company opened did they save $10,000$?

   A. 1
   
   B. 4
   
   C. 5
   
   D. 6

2. A computer help service charges an initial fee to join the service plus an additional charge for each hour of help service a customer uses. If the computer service company charges a total of $140$ for the initial fee and a 2-hour help session and a total of $220$ for the initial fee and a 4-hour help session, which of the following expressions gives the computer company’s charge for each hour of help-service that a customer uses?

   A. $\frac{220-140}{4-2}$
   
   B. $\frac{220+140}{4+2}$
   
   C. $\frac{4-2}{220-140}$
   
   D. $\frac{4+2}{220+140}$

3. Jen scored 16 points in a new card game, where each player could receive either 2 or 4 points in each round. If Jen received $x$ amount of 2 point scores, and $y$ amount of 4 point scores, what does the $x$-intercept of the graph in the $xy$-plane of the equation $2x + 4y = 16$ indicate?

   A. Jen scored 2 points in 8 rounds and she didn’t score 4 points in any round.
   
   B. Jen scored 2 points in 2 rounds and 4 points in 3 rounds.
   
   C. Jen scored 2 points in 4 rounds and 4 points in 2 rounds.
   
   D. Jen didn’t score 2 points in any round, but she scored 4 points in 4 rounds.

4. Which of the following is true about the line graphed in the $xy$-plane above?

   A. The line has slope $\frac{2}{3}$ and $y$-intercept $-3$.
   
   B. The line has slope $\frac{2}{3}$ and $y$-intercept $2$.
   
   C. The line has slope $\frac{3}{2}$ and $y$-intercept $-3$.
   
   D. The line has slope $\frac{3}{2}$ and $y$-intercept $2$. 
POLYNOMIALS AND QUADRATIC APPLICATIONS

For each of the questions below, choose the best answer from the four choices given. You may use the paper you received as scratch paper.

1. \((xy^3z^4)(x^{-4}y^{-3}z^{-1}) = \)
   A. \(\frac{z}{x}\)
   B. \(\left(\frac{z}{x}\right)^3\)
   C. \(y\left(\frac{z}{x}\right)\)
   D. \(y\left(\frac{z}{x}\right)^3\)

2. \(\left(\frac{a}{2} - b\right)^2 = \)
   A. \(\frac{a^2}{2} - ab + b^2\)
   B. \(\frac{a^2}{2} - 2ab + b^2\)
   C. \(\frac{a^2}{4} - ab + b^2\)
   D. \(\frac{a^2}{4} - 2ab + b^2\)

3. If \(x^2 - 3x - 18 = 0\), which of the following is a possible value for \(x\)?
   A. -6
   B. 3
   C. 6
   D. 9

4. The function \(f(x) = -x^2 + 40x - 175\) is graphed in the \(xy\)-plane above. For what value of \(x\) is the value of \(f(x)\) greatest?
   A. 5
   B. 20
   C. 30
   D. 35

5. \((x + 9)\left(\frac{1}{x^2+2x-63}\right) = \)
   A. \(x - 7\)
   B. \(x + 7\)
   C. \(\frac{1}{x-7}\)
   D. \(\frac{1}{x+7}\)
## Answer Key

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Correct Answer</th>
<th>Rationale</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>C</td>
<td>Choice (C) is correct. The low temperature of 75°F was reached at 6 in the morning, and the high temperature was 16°F higher. So the high temperature in Benton that day was 75°F + 16°F = 91°F.</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td><img src="image" alt="Number Line" /> Choice (A) is correct. The absolute value of point A is</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>Choice (A) is correct. The value of -2 - 4 is -2 + (-4) = -6. Therefore, (-2 - 4) x 8 = -6 x 8 = -48.</td>
</tr>
<tr>
<td>4</td>
<td>C</td>
<td>Choice (C) is correct. Cheryl’s average score for the five quizzes will be the sum of the scores divided by 5, the number of quizzes. She scored a total of 364 points on the first four quizzes, and if she scores 96 points on her next quiz, the sum of the scores will be 364 + 96 = 460 points. Therefore, her average score for the five quizzes will be 460 ÷ 5 = 92 points.</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>Choice (B) is correct. The square root of 529, denoted √529, is 23, because 23² = 23 x 23 = 529.</td>
</tr>
</tbody>
</table>
**Fractional and Decimals**

<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>B</td>
<td>Choice (B) is correct. Since the table is a semicircle of diameter 12 feet, the radius of the semicircle is 6 feet. The area of the table is ( \frac{1}{2} \times \pi \times 6^2 ) square feet, or approximately ( 18 \times 3.14 = 56.52 ) square feet. Therefore, of the choices given, the closest to the area of the table is choice (B), 57 square feet.</td>
</tr>
<tr>
<td>2</td>
<td>D</td>
<td>Choice (D) is correct. Since the large square has area 9, each of its sides is of length 3. Hence each of the 9 smaller squares has sides of length 1. Since the path drawn in bold is made up of six of the sides of smaller squares, its length is ( 6 \times 1 = 6 ).</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>Choice (A) is correct. The division ( 0.6 \div 10^{-2} ) is equivalent to the multiplication ( 0.6 \times \frac{1}{10^{-2}} = 0.6 \times 10^2 = 0.6 \times 100 = 60 ).</td>
</tr>
<tr>
<td>4</td>
<td>C</td>
<td>Choice (C) is correct. The number 3,590 is equal to the product ( 3.59 \times 1,000 ), which can be rewritten as ( 3.59 \times 10^3 ).</td>
</tr>
</tbody>
</table>
# Answer Key

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<tr>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>Choice (A) is correct. Since 40% of 25 is ( \frac{40}{100} \times 25 = 10 ), and 52% of 25 is ( \frac{52}{100} \times 25 = 13 ). Jeron made 3 more free-throw attempts than Sidell did.</td>
</tr>
<tr>
<td>2</td>
<td>D</td>
<td>Choice (D) is correct. Since there 60 minutes in an hour, the 10-minute interval is equivalent to ( \frac{1}{6} ) of an hour. The boy’s average speed can be calculated as number of miles skied ( \frac{\text{time}}{\text{time}} ), which is ( \frac{4}{\frac{5}{6}} = 4 \times \frac{6}{1} = 24 ) miles per hour.</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>Choice (C) is correct. Since every 4th child in line, starting with the fourth in line, gets a toy, it follows that the children who get a toy are in line in positions 4, 8, 12, 16 and 20. Hence, of the 23 children in the line, 5 get a toy, and 23 – 5 = 18 do not get a toy. Therefore, the ratio of the number of children in line who get a toy to the number of children in line who do not get a toy is 5:18.</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>Choice (B) is correct. Since ( \frac{52}{160} = \frac{13}{40} = 0.325 ), it follows that 52 is 32.5 percent of 160.</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>Choice (B) is correct. Since there are about 1.6 kilometers in 1 mile, it follows that Jenna’s speed in kilometers per hour is ( 65 \times 1.6 = 104 ) kilometers per hour.</td>
</tr>
</tbody>
</table>
**Answer Key**

<table>
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</table>
| 1               | C             | ![Number Line](image)  
Choice (C) is correct. The value of \(x\) on the number line is \(-\frac{2}{3}\), and the value of \(y\) on the number line is 2. Substituting these values into the expression \(y^2 + 2x\) gives  
\[
2^2 + 2 \left( -\frac{2}{3} \right) = 4 - \frac{4}{3} = \frac{8}{3}.
\]  
| 2               | B             | Choice (B) is correct. The rental fee for the costume consists of the initial charge of \$20\ and a daily charge of \$8\. Thus if the costume is rented for \(n\) days, the total cost, in dollars, is \(20 + 8n\). |
| 3               | D             | Choice (D) is correct. If the original cost of the treadmill is \(t\) dollars, an 8% discount on that price is \(0.08t\) dollars. Therefore, the discounted price is the original price, in dollars, minus the discount, which is \(t - 0.08t\). |
| 4               | B             | Choice (B) is correct. The equation \(\frac{x}{3} - 2 = 5x - 2\) is equivalent to \(\frac{x}{3} = 5x\). Multiplying both sides of this equation by 3 gives \(15x = x\). Subtracting \(x\) on each side of the equation yields \(14x = 0\), so \(x = 0\). |
# Answer Key

## Graphs and Equations of Lines

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<th>Rationale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C</td>
<td>Choice (C) is correct. The ( y )-value represents the total amount of money that the company saved. From the graph, after 5 years the company was opened, they saved $10,000.</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>Choice (A) is correct. The expression ( \frac{220-140}{4-2} ) represents the difference of dollars charged for two different help sessions divided by the difference in the number of hours of help-service used, giving the amount, in dollars, the company charges for each hour of help-service a customer uses.</td>
</tr>
</tbody>
</table>
## Answer Key

### Graphs and Equations of Lines

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<tbody>
<tr>
<td>3</td>
<td>A</td>
<td>Choice (A) is correct. Since $y$ represents the amount of 4 points Jen scored, and the $x$-intercept is the value of $x$ that satisfies equation $2x + 4y = 16$, when $y = 0$, Jen must have scored only 2 points each round. Since she scored a total of 16 points, she must have scored 2 points in 8 rounds.</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>Choice (B) is correct. The $y$-value of the line increases 2 units for every 3 units of increase in the $x$-value. Therefore, the slope of the line is $\frac{2}{3}$. The line also intersects the $y$-axis at 2, and therefore the $y$-intercept is 2.</td>
</tr>
</tbody>
</table>
## Answer Key

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<tbody>
<tr>
<td>1</td>
<td>B</td>
<td>Choice (B) is correct. By the law of exponents that ((xy^3z^4)(x^{-4}y^{-3}z^{-1}) = x^{(1-4)}y^{(3-3)}z^{(4-1)}). Therefore, ((xy^3z^4)(x^{-4}y^{-3}z^{-1}) = x^{-3}y^0z^3). This is equivalent to (\frac{x^3}{x^3} = \left(\frac{z}{x}\right)^3).</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>Choice (C) is correct. By definition, the expression (\left(\frac{a}{2} - b\right)^2) is (\left(\frac{a}{2} - b\right)\left(\frac{a}{2} - b\right)). This expression is equivalent to (\left(\frac{a}{2}\right)^2 - b \left(\frac{a}{2}\right) - \left(\frac{a}{2}\right) b + b^2). It follows that (\left(\frac{a}{2} - b\right)^2) is equivalent to (\left(\frac{a}{2}\right)^2 - \frac{ab}{2} - \frac{ab}{2} + b^2), which simplifies to (\frac{a^2}{4} - ab + b^2).</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>Choice (C) is correct. The expression (x^2 - 3x - 18) factors as ((x - 6)(x + 3)). Since (x^2 - 3x - 18 = 0), either (x - 6 = 0) or (x + 3 = 0). It follows that (x = 6) or (x = -3). Of the options given, only 6 is a possible value for (x).</td>
</tr>
</tbody>
</table>
### Answer Key

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</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>B</td>
<td>Choice (B) is correct. Since ( f(20) = 225 ) is larger than any other function value, the function is maximized at ( x = 20 ).</td>
</tr>
<tr>
<td>5</td>
<td>C</td>
<td>Choice (C) is correct. ( (x + 9) \left( \frac{1}{x^2 + 2x - 63} \right) = \frac{x + 9}{(x + 9)(x - 7)} = \frac{1}{x - 7} )</td>
</tr>
</tbody>
</table>
## FRACTIONS

**Method:**

**To simplify a fraction,** divide the numerator and denominator by all common factors.

**Examples:**

\[
\frac{12}{18} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}
\]

**To multiply fractions:**

1. Divide out all factors common to a common numerator and any denominator.
2. Multiply numerators.
3. Multiply denominators.

**Examples:**

\[
\frac{5}{6} \times \frac{4}{15} = \frac{5 \times 4}{6 \times 15} = \frac{2}{9}
\]

**To divide fractions,** invert the second fraction and multiply.

**Examples:**

\[
\frac{5}{6} \div \frac{1}{2} = \frac{5}{6} \times \frac{2}{1} = \frac{5}{3} \times \frac{2}{1} = \frac{5}{3}
\]

**To add or subtract fractions:**

1. Find the Least Common Denominator (LCD)
2. In each fraction, multiply the numerator and denominator by the same number to obtain the common denominator.
3. Add or subtract the numerators and keep the common denominator.

**Examples:**

\[
\frac{1}{2} + \frac{3}{7} + \frac{5}{8} = \frac{1 \times 28}{2 \times 28} + \frac{3 \times 8}{7 \times 8} + \frac{5 \times 7}{8 \times 7} = \frac{28}{56} + \frac{24}{56} + \frac{35}{56} = \frac{87}{56}
\]

**To change a mixed number to an improper fraction:**

1. Multiply the denominator by the whole number.
2. Add the product to the numerator.
3. Place the sum over the denominator.

**Examples:**

\[
5 \frac{2}{3} = \frac{3 \times 5 + 2}{3} = \frac{17}{3}
\]
FRACTIONS (CONT'D)

<table>
<thead>
<tr>
<th>Method:</th>
<th>Examples:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>To change an improper fraction to a mixed number:</strong></td>
<td>Ex: $\frac{42}{5} = 8\frac{2}{5}$ since $5\times\frac{42}{5} = 40\frac{2}{5}$</td>
</tr>
<tr>
<td>1. Divide the denominator into the numerator.</td>
<td></td>
</tr>
<tr>
<td>2. The whole number in the mixed number is the quotient, and the fraction is the remainder over the denominator.</td>
<td></td>
</tr>
<tr>
<td><strong>To change a whole number to a fraction,</strong> write the number over 1.</td>
<td>Ex: $9 = \frac{9}{1}$</td>
</tr>
<tr>
<td><strong>To multiply or divide the whole numbers and/or mixed numbers:</strong></td>
<td>Ex: $2\frac{1}{3} + 5$</td>
</tr>
<tr>
<td>1. Change to improper fractions.</td>
<td></td>
</tr>
<tr>
<td>2. Multiply or divide the fractions.</td>
<td>$= \frac{7}{3} \div \frac{5}{1} = \frac{7}{3} \times \frac{1}{5} = \frac{7}{15}$</td>
</tr>
<tr>
<td>3. Change the answer to mixed number.</td>
<td></td>
</tr>
<tr>
<td><strong>To add mixed numbers,</strong> change to improper fractions, add, and then convert to a mixed number.</td>
<td>Ex: $6\frac{3}{4} + 2\frac{5}{8}$</td>
</tr>
<tr>
<td>OR Add the whole numbers and fractions separately.</td>
<td>Ex: $\frac{6\frac{3}{4}}{8} + \frac{6\frac{5}{8}}{8} = 8 + 1\frac{3}{8} = 9\frac{3}{8}$</td>
</tr>
<tr>
<td>If an improper fraction results, change it to a mixed number and add the whole numbers.</td>
<td></td>
</tr>
<tr>
<td><strong>To subtract mixed numbers,</strong> change to improper fractions, subtract, and convert to mixed numbers.</td>
<td>Ex: $8\frac{1}{5} - 4\frac{2}{3}$</td>
</tr>
<tr>
<td>OR Subtract the whole numbers and fractions separately.</td>
<td>Ex: $-\frac{4\frac{3}{5}}{15} - 4\frac{10}{15} = -\frac{7\frac{18}{15}}{3\frac{3}{15}}$</td>
</tr>
<tr>
<td>If necessary, borrow a fraction equal to 1 from the whole number.</td>
<td></td>
</tr>
</tbody>
</table>
## DECIMALS

### Method:

<table>
<thead>
<tr>
<th>To determine which of two decimals is larger:</th>
<th>Examples:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Write the decimals so that they have the same number of digits (by adding zeros).</td>
<td>Ex: (.257 &lt; .31) since (.257 &lt; .310)</td>
</tr>
<tr>
<td>2. Start at the left and compare corresponding digits. The larger number will have the larger digit.</td>
<td>and (2 &lt; 3)</td>
</tr>
</tbody>
</table>

### To round a decimal:

<table>
<thead>
<tr>
<th>1. Locate the place for which the round off is required.</th>
<th>Ex: Round: (1.5725) to the</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Compare the first digit to the right of this place to 5. If this digit is less than 5, drop it and all digits to the right of it. If this digit is &quot;greater than or equal to 5,&quot; increase the rounded digit by one and drop all digits to the right.</td>
<td>a) (1.5725 = 1.57)</td>
</tr>
<tr>
<td>--------------------------------------------------------</td>
<td>b) (1.5725 = 1.573)</td>
</tr>
</tbody>
</table>

### To add or subtract decimals:

<table>
<thead>
<tr>
<th>1. Write the numbers vertically and line up the decimal points. If needed, add zeros to the right of decimal digits.</th>
<th>Ex: Add:</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Add or subtract as with whole numbers.</td>
<td>3.65 + 12.2 + .51</td>
</tr>
<tr>
<td>3. Align the decimal point in the answer with the other decimal points.</td>
<td>3.65</td>
</tr>
<tr>
<td></td>
<td>12.20</td>
</tr>
<tr>
<td></td>
<td>(+ .51)</td>
</tr>
<tr>
<td></td>
<td>16.36</td>
</tr>
</tbody>
</table>

### To multiply decimals:

<table>
<thead>
<tr>
<th>1. Multiply the numbers as whole numbers.</th>
<th>Ex: Multiply:</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Determine the sum of decimal places in the 2 numbers.</td>
<td>(.0023 \times .14)</td>
</tr>
<tr>
<td>3. Make sure the answer has the same number of decimal places as the sum from Step 2. (Insert zeros to the left if necessary.)</td>
<td>(.0023 \times .14 = .000322) 6 decimal places total</td>
</tr>
<tr>
<td></td>
<td>(092)</td>
</tr>
<tr>
<td></td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>(.000322) 6 decimal places in result</td>
</tr>
</tbody>
</table>
**DECIMALS CONT'D**

**Method:**

**To divide decimals:**

1. Make the divisor a whole number by moving the decimal point to the right. (Mark this position with a caret \(^\wedge\).) 

2. Move the decimal in the dividend to the right the same number of places. (Mark this position with a caret \(^\wedge\).) 

3. Place the decimal point in the answer directly above the caret. 

4. Divide as with whole numbers, adding zeros to the right if necessary. Continue until the remainder is zero, the decimal digits repeat, or the desired number of decimal positions is achieved. 

**Examples:**

Ex: Divide: 

\[
\begin{array}{c|c}
& 3.36 \\
.05 & \overline{0.1680} \\
\end{array}
\]

Ex: 

\[
\begin{array}{c|c}
15 & 15 \\
18 & 30 \\
\hline \\
30 & 0 \\
\end{array}
\]

**To convert a fraction to a decimal,** divide the denominator into the numerator. 

**Examples:**

Ex: Convert: 

\[
\frac{9}{11} \text{ to a decimal}
\]

\[
\frac{0.8181...}{11} \div 9.0000 = 0.81
\]

**PERCENTS**

**Technique:**

% means "per 100" or "out of 100"

**Examples:**

Ex: 45% means \(\frac{45}{100}\) or 45 out of 100.

Ex: 100% is equal to \(\frac{100}{100}\) or 1

**To convert a percentage to a fraction or decimal,** divide by 100%. 

**Examples:**

Ex: Convert: 32% to a fraction.

\[
32\% = \frac{32\%}{100\%} = \frac{32}{100} = \frac{8}{25}
\]

Ex: Convert: 25% to a decimal.

\[
25\% = \frac{25\%}{100\%} = \frac{25}{100} = 0.25
\]
### PERCENTS CONT'D

<table>
<thead>
<tr>
<th>Method:</th>
<th>Examples:</th>
</tr>
</thead>
</table>
| **To convert a fraction or decimal to a percentage,** multiply by 100%. | Ex: Convert: \( \frac{3}{5} \) to a percent.  
\[
\frac{3}{5} \times 100\% = \frac{3}{5} \times \frac{100^{20}}{1} \% = 60\%
\]
| **Note:** A shortcut for multiplying by 100 is moving the decimal 2 places to the right | Ex: Convert: 1.4 to a percent  
1.4 \times 100\% = 140\% |
| **To calculate a percentage of a quantity**. convert the percentage to a decimal or fraction, then multiply the result by the given quantity. | Ex: What is 20% of 80?  
20\% = .20 and  
.20 \times 80 = 16  
OR  
20\% = \frac{1}{5} and  
\[
\frac{1}{5} \times 80 = 16
\]
so 20\% of 80 is 16. |
| **To solve percent equations:**  
1. Change the percent to a decimal or fraction. | Ex: 14 is 25\% of what number?  
14 = .25x  
14 = \frac{25x}{.25}  
.25 .25  
56 = x  
so 14 is 25\% of 56 |
| 2. Translate the question to an equation by replacing "is" with =, "of" with multiplication, and "what" with a variable. |  
| 3. Solve the equation for the variable. |  
| **To solve a percent increase or decrease problem,** use the following models: | Ex: If 68,000 was increased to 78,500, find the percent increase.  
Increase  
new = original + percent as decimal \times original  
78,500 = 68,000 + x(68,000)  
10,500 = 68,000x  
10,500 = 68,000x  
68,000 \times 68,000  
\[.154 = x\] So the percent increase is 15.4\%  
Decrease  
new = original - percent as decimal \times original
## VARIOUS APPLICATIONS

**Technique:**  
**To convert units**, multiply by a fraction consisting of a quantity divided by the same quantity with different units (multiplication by 1). Set up the fraction so that the units will cancel appropriately.

**Examples:**  
**Ex:** Convert 5 meters to feet using the fact that 1 ft = .305 meters  

\[
1 \text{ ft} = 0.305 \text{ meters}  
5 \text{ meters} \times \frac{1 \text{ ft}}{0.305 \text{ meters}} = \frac{5 \text{ ft}}{0.305} = 16.39 \text{ ft}
\]

**To solve problems involving vehicle travel**, use Distance = Rate × Time.  
**Note:** Rate is the average speed.

**Ex:** How long does it take a car traveling 55 mph to travel 30 miles?  
\[
d = rt  
30 = 55t  
\frac{30}{55} = t  
t = 0.55 \text{ hours or 33 minutes}
\]

**To solve problems involving sides of a right triangle**, use the Pythagorean Theorem. In a right triangle, if \(a\) and \(b\) are the legs and \(c\) is the hypotenuse, \(a^2 + b^2 = c^2\).

**Ex:** A plane flew in a straight line to a point 100 miles west and 150 miles north from where it began. How far did that plane travel?  
\[
c^2 = 100^2 + 150^2  
c^2 = 32500  
c = \sqrt{32500}  
c = 180.28 \text{ miles}
\]

The plane flew 180.28 miles

**To find an average of several values**, add the values and divide the total by the number of values.

**Ex:** The average value of 15, 21, and 24 is 20, because \[
\frac{15 + 21 + 24}{3} = \frac{60}{3} = 20
\]
<table>
<thead>
<tr>
<th>Verbal Description</th>
<th>Algebraic Operation or Symbol</th>
<th>Examples:</th>
</tr>
</thead>
<tbody>
<tr>
<td>is, equal, are, results in</td>
<td>equal sign</td>
<td>Ex: a number plus 7 results in 10 $x + 7 = 10$</td>
</tr>
<tr>
<td>sum, plus, increased by, greater than, more than, exceeds, total of</td>
<td>addition</td>
<td>Ex: the sum of a number and 2 $x + 2$</td>
</tr>
<tr>
<td>difference, minus, decreased by, less than, subtracted from, reduced by, the remainder</td>
<td>Subtraction</td>
<td>Ex: 7 subtracted from 5 $5 - 7$</td>
</tr>
<tr>
<td>product, multiplied by, twice, times, of</td>
<td>multiplication</td>
<td>Ex: twice a number $2x$</td>
</tr>
<tr>
<td>quotient, divided by, ratio, per</td>
<td>division</td>
<td>Ex: 35 miles per hour $\frac{35 \text{ miles}}{1 \text{ hour}}$</td>
</tr>
<tr>
<td>exponent, power, squared, cubed</td>
<td>exponent</td>
<td>Ex: two cubed $2^3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ex: a number to the 5th power $x^5$</td>
</tr>
<tr>
<td>ratio</td>
<td>colon or fraction</td>
<td>Ex: the ratio of 1 to 4 is 1:4 or $\frac{1}{4}$</td>
</tr>
</tbody>
</table>

**Note:** Parentheses must be used to indicate an operation is to be applied to an entire expression.

**Note:** Use a variable to represent an unknown quantity.
# BASIC DEFINITIONS

<table>
<thead>
<tr>
<th>Definitions:</th>
<th>Examples:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Integers</strong> = all positive and negative whole numbers and zero.</td>
<td>Ex: -100, 20, 0, -451 (all integers)</td>
</tr>
<tr>
<td><strong>Rational Numbers</strong> = all terminating or repeating decimals.</td>
<td>Ex: ( \frac{1}{4} = .25 ) and ( \frac{2}{3} = .\overline{6} ) are rational</td>
</tr>
<tr>
<td><strong>Irrational Number</strong> = all nonterminating, nonrepeating decimals.</td>
<td>Ex: ( \sqrt{2} = 1.4142135\ldots ) is irrational. Ex: ( \pi = 3.1415926\ldots ) is irrational.</td>
</tr>
<tr>
<td><strong>Prime Number</strong> = positive integer greater than 1 with no integer factors other than itself and 1.</td>
<td>Ex: 5, 17, 29, and 37 (all prime)</td>
</tr>
</tbody>
</table>

### Real Number Line

Ex: Plot \( \frac{5}{3} \) on the number line. \( \frac{5}{3} = 1 \frac{2}{3} \)

### Absolute Value

| \(|a|\) = the distance between \(a\) and 0 on the number line. | Ex: \(|2| = 2\) \(|0| = 0\) \(|-3| = 3\) |

To determine absolute value, just make the number positive.

### Square Root

| \(\sqrt{a}\) = the positive number that can be multiplied by itself to yield \(a\). | Ex: \(\sqrt{25} = 5\) \(\sqrt{100} = 10\) |
**OPERATIONS ON INTEGERS**

<table>
<thead>
<tr>
<th>Techniques:</th>
<th>Examples:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition</strong></td>
<td><strong>Examples:</strong></td>
</tr>
</tbody>
</table>
| If the numbers have the same signs, add the absolute value and attach the common sign to the result. | Ex: 5 + 12 = 17  
Ex: -4 + (-10) = -14  
Ex: (-3) + (-7) + (-10) = -20 |
| If the numbers have opposite signs, subtract the smaller absolute value from the larger and attach the sign of the larger. | Ex: 5 + (-12) = -7  
Ex: (-7) + 3 = -4  
Ex: -2 + 10 = 8 |
| **Subtraction** | **Examples:** |
| Add the opposite of the second number. | Ex: 7 - 15 = 7 + (-15) = -8 |
| Subtracting a positive is the same as adding a negative. | Ex: 4 - 5 = 4 + (-5) = -1 |
| Subtracting a negative is the same as adding a positive. | Ex: 7 - (-3) = 7 + 3 = 10 |
| **Multiplication and Division** | **Examples:** |
| If the numbers have the same signs, perform the operation on the absolute values and attach a positive sign to the result. | Ex: (-6) ÷ (-2) = 3  
Ex: \(-\frac{12}{4}\) = 3  
Ex: (-3)(-2) = 6 |
| If the numbers have opposite signs, perform the operation on the absolute values and attach a negative sign to the result. | Ex: (-5)(2) = -10  
Ex: \(-\frac{15}{3}\) = -5 |
| **Exponents (positive integer exponents)** | **Examples:** |
| Multiply the base the number of times given by the exponent. | Ex: \((-2)^3\) = \((-2)(-2)(-2)\) = -8  
Ex: \(2^5\) = \((2)(2)(2)(2)(2)\) = 32  
Ex: \((-2)^4\) = \((-1)(2)(2)(2)(2)\) = -16  
Ex: \((-2)^4\) = \((-2)(-2)(-2)(-2)\) = 16 |
**ORDER OF OPERATIONS**

**Techniques:**

<table>
<thead>
<tr>
<th>Techniques</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>To evaluate an expression using the order of operations,</strong> perform operations in the following order:</td>
<td>Ex: Evaluate (16 \div 2^3 - 4(3 -</td>
</tr>
</tbody>
</table>

1. **Parentheses**
   - Starting with the innermost symbol, perform operations inside symbols of grouping (parentheses or brackets) or absolute value symbols.
   
   \[
   = 16 \div 2^3 - 4(3 - |5 - 7|) + 5 \\
   = 16 \div 2^3 - 4(3 - |-2|) + 5 \\
   = 16 \div 2^3 - 4(3 - 2) + 5 \\
   = 16 \div 2^3 - 4(1) + 5
   \]

2. **Exponents**
   - Evaluate all exponential expressions.
   
   \[
   = 16 \div 8 - 4(1) + 5
   \]

3. **Multiplication/Division**
   - In order from left to right, perform all multiplications and divisions.
   
   \[
   = 2 - 4(1) + 5 \\
   = 2 - 4 + 5
   \]

4. **Addition/Subtraction**
   - In order from left to right, perform all additions and subtractions.
   
   \[
   = -2 + 5 \\
   = 3
   \]
## Evaluating Algebraic Expressions

### Techniques:
To evaluate an expression at given values of the variables:

### Examples:

**Ex:** Evaluate \( y^2 - x + 2z + z^3 \) when: 
\[ x = -3, \ y = -5 \text{ and } z = 2 \]

1. Replace every occurrence of each variable with the appropriate real number. Use parentheses when substituting a negative number for a variable.

\[ y^2 - x + 2z + z^3 = (-5)^2 - (-3) + 2(2) + 2^3 \]

2. Use the order of operations to evaluate the resulting expression.

\[ = 25 + 3 + 4 + 8 \]
\[ = 28 + 12 \]
\[ = 40 \]

## Simplifying Algebraic Expressions

### Techniques:
To simplify an algebraic expression:

### Examples:

**Ex:** Simplify:
\[ (5x)^2 + 4 [x^2 - (2x - 5)] \]

1. Starting with the innermost set, remove symbols of grouping. Usually the distributive property and/or rules of exponents must be used in this step.

\[ = (5x)^2 + 4 [x^2 - 2x + 5] \]
\[ = 25x^2 + 4x^2 - 8x + 20 \]

2. Combine like terms by adding the coefficients of terms having the same variable factor.

\[ = 29x^2 - 8x + 20 \]
**PERIMETER, AREA, AND VOLUME**

### Formulas:

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>( A = w^2 )</td>
<td>5 in 5 in ( A = 25 \text{in}^2 )</td>
</tr>
<tr>
<td>Rectangle</td>
<td>( A = lw )</td>
<td>6 cm 7 cm ( A = 42 \text{cm}^2 )</td>
</tr>
<tr>
<td>Parallelogram</td>
<td>( A = bh )</td>
<td>6 ft 5 ft 10 ft ( A = 10(5) = 50 \text{ft}^2 )</td>
</tr>
<tr>
<td>Triangle</td>
<td>( A = \frac{1}{2} bh )</td>
<td>7 in 6 in 12 in ( A = \frac{1}{2} (12)(6) = 36 \text{in}^2 )</td>
</tr>
<tr>
<td>Circle</td>
<td>( A = \pi r^2 ) ( \pi = 3.14 ) ( A = \pi(4)^2 = 50.27 \text{in}^2 ) ( C = 2\pi r ) ( C = 2\pi(4) = 25.13 \text{in} )</td>
<td></td>
</tr>
<tr>
<td>Any Polygon</td>
<td>( P = \text{sum of the sides} )</td>
<td>6 in 6 in 10 in 13 in ( P = 6 + 6 + 13 + 10 = 35 \text{in} )</td>
</tr>
</tbody>
</table>
### PERIMETER, AREA, AND VOLUME CONT'D

<table>
<thead>
<tr>
<th>Formulas:</th>
<th>Examples:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cube</strong></td>
<td>$V = w^3$</td>
</tr>
<tr>
<td>5 cm</td>
<td>5 cm</td>
</tr>
<tr>
<td><strong>Rectangular Solid</strong></td>
<td>$V = lwh$</td>
</tr>
<tr>
<td>4 in</td>
<td>9 in</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Cylinder</strong></td>
<td>$V = \pi r^2 h$</td>
</tr>
<tr>
<td>2 in radius</td>
<td>4 in height</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Sphere</strong></td>
<td>$V = \frac{4}{3} \pi r^3$</td>
</tr>
<tr>
<td>3 in</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
LINEAR EQUATIONS

Techniques:  

To solve a linear equation:  

Ex:  Solve \( \frac{2}{3} (x + 2) + \frac{5}{6} x = x + 4 \)

1. Remove fractions by multiplying both sides by the least common denominator (LCD).

   LCD = 6 so multiply each term by 6 on both sides

   \[
   6 \cdot \frac{2}{3} (x + 2) + 6 \cdot \frac{5}{6} x = 6(x + 4)
   \]

2. Simplify each side by distributing and combining like terms.

   \[
   4(x + 2) + 5x = 6(x + 4)
   \]

3. Add or subtract variable terms from both sides so that the variable will occur on only one side.

   \[
   9x + 8 = 6x + 24
   \]

4. Isolate the variable by performing the same operation on both sides of the equation.

   \[
   3x + 8 = 24
   \]

   \[
   3x = 16
   \]

   \[
   x = \frac{16}{3}
   \]

   \[
   \text{Note: A check is necessary if the equation contains fractions with variable denominators.}
   \]

To solve an application problem using a linear equation:  

Ex:  A person has 90 coins in quarters and dimes with a combined value of $16.80. Determine the number of coins of each type.

1. Write a verbal equivalence containing the quantities involved in the problem.

   Value of quarters + value of dimes = total value

2. Substitute given values for known quantities.

   \[
   .25 \text{(number of quarters)} + .10 \text{(number of dimes)} = \text{total value}
   \]

   \[
   .25 \text{(number of quarters)} + .10 \text{(number of dimes)} = $16.80
   \]

3. Use a variable to represent one unknown quantity in the equation.

   \[
   .25x + .10 \text{(number of dimes)} = 16.80
   \]

4. Replace the remaining unknown quantities with an appropriate expression involving the variable.

   \[
   .25x + .10(90 - x) = 16.80
   \]

5. Solve the equation.

   \[
   .25x + 9 - .10x = 16.80
   \]

   \[
   .15x + 9 = 16.80
   \]

   \[
   15x = 7.8
   \]

   \[
   x = 52
   \]

6. Answer the original question.

   \[
   x = 52 \quad 90 - x = 90 - 52 = 38
   \]

   There are 52 quarters and 38 dimes.
**INEQUALITIES**

**Techniques:**

To solve a linear inequality, employ the same procedure used for solving equations, but remember to reverse the order symbol when multiplying or dividing both sides by a negative number.

**Examples:**

Ex: Solve \(-2x < 20\)

\[-2x < 20\]

\[-2 \cdot -2 \text{ (divided by negative)}\]

\[x > -10 \text{ (reverse direction of symbol)}\]

To solve a compound inequality, isolate the variable in the center by performing the same operation on all three parts.

Ex: Solve \(-13 \leq 6x - 1 < 3\)

\[-13 \leq 6x - 1 < 3\]

\[+1 +1 +1\]

\[-12 \leq 6x < 4\]

\[6 6 6\]

\[-2 \leq x < \frac{2}{3}\]

To graph an inequality:

1. Isolate the variable.

Ex: Graph \(x + 2 < 1\)

\[x + 2 < 1\]

\[-2 -2\]

\[x < -1\]

2. Shade in the numbers on the real number line that satisfy the inequality.

3. Use closed dots or brackets to show the endpoint is included with \(\leq\) or \(\geq\). Use open dots or parentheses to show the endpoint is NOT included with \(<\) or \(>\).

To give the solution set of inequality in interval notation:

1. Isolate the variable.

Ex: Write \(-6 \leq x < 3\) in interval notation

\([-6, 3)\]

2. List smallest number and largest number in the solution set separated with a comma. If no smallest number, use \(-\infty\). If no largest number, use \(\infty\).

Ex: Write \(x < 2\) in interval notation

\((-\infty, 2)\)

\((-\infty \text{ is used since there is no smallest number that is less than } 2\).
## EXPONENTS

### Rules:

<table>
<thead>
<tr>
<th><strong>Examples:</strong></th>
<th><strong>A positive integer exponent</strong> dictates the number of times the base is to be multiplied.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex:</td>
<td>$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$</td>
</tr>
<tr>
<td>Ex:</td>
<td>$(-1)^5 = (-1)(-1)(-1)(-1)(-1)$</td>
</tr>
<tr>
<td></td>
<td>$= (1) \cdot (1) \cdot (-1)$</td>
</tr>
<tr>
<td></td>
<td>$= -1$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Examples:</strong></th>
<th><strong>A negative integer exponent</strong> applied to a base is equal to the reciprocal of the base raised to the opposite exponent.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex:</td>
<td>$2^{-3} = \frac{1}{2^3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8}$</td>
</tr>
<tr>
<td>Ex:</td>
<td>$x^{-5} = \frac{1}{x^5}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Examples:</strong></th>
<th><strong>The zero exponent</strong> applied to any base (except 0) is equal to 1.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex:</td>
<td>$4^0 = 1$</td>
</tr>
<tr>
<td>Ex:</td>
<td>$(-12)^0 = 1$</td>
</tr>
<tr>
<td>Ex:</td>
<td>$-(12)^0 = -1$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Examples:</strong></th>
<th><strong>A factor can be moved from numerator to denominator (or vice versa) by changing the sign of the exponent.</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex:</td>
<td>$\frac{3^2 \cdot 4^{-2}}{2^{-3}} = \frac{3^2 \cdot 2^3}{4^2} = \frac{9 \cdot 8}{16} = \frac{9}{2}$</td>
</tr>
<tr>
<td>Ex:</td>
<td>$\frac{3xy^{-3}}{z^{-4}} = \frac{3xz^4}{y^3}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Examples:</strong></th>
<th><strong>An exponent can be applied to each part of a product or fraction.</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex:</td>
<td>$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$</td>
</tr>
<tr>
<td>Ex:</td>
<td>$(3xy)^2 = 3^2x^2y^2 = 9x^2y^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Examples:</strong></th>
<th><strong>To apply an exponent to a sum or difference, multiply the polynomial by itself.</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex:</td>
<td>$(x + 1)^2 = (x + 1)(x + 1)$</td>
</tr>
<tr>
<td></td>
<td>$= x^2 + 2x + 1$</td>
</tr>
</tbody>
</table>
**OPERATIONS ON EXPONENTS**

**Procedures:**

**Examples:**

- **To multiply expressions with the same base,** keep the base and add the exponents.
  
  \[ x^2 \cdot x^4 = x^{2+4} = x^6 \]

- **To divide expressions with the same base,** keep the base and subtract the exponents.
  
  \[ \frac{y^9}{y^5} = y^{9-5} = y^4 \]

- **To raise an exponential expression to another power,** keep the base and multiply the exponents.
  
  \[ (x^3)^2 = x^{3\cdot2} = x^6 \]

- **To add or subtract expressions with exponents,** combine like terms. The base and the exponent must be the same.
  
  \[ 3x^2 + 5x^2 = 8x^2 \]

**OPERATIONS ON POLYNOMIALS**

**Techniques:**

**Examples:**

- **To add polynomials,** combine like terms (terms with the same variable raised to the same exponent.)

  \[ (4x^3 - 2x^2 - 7x) + (-6x^3 - 3x^2 + 5) \]
  
  \[ = 4x^3 - 2x^2 - 7x - 6x^3 - 3x^2 + 5 \]
  
  \[ = -2x^3 - 5x^2 - 7x + 5 \]

- **To subtract polynomials:**
  
  1. Distribute the −1. (This will change the sign of each term on the 2nd polynomial.)
  
  \[ (4x^3 - 2x^2 - 7x) - (-6x^3 - 3x^2 + 5) \]
  
  \[ = 4x^3 - 2x^2 - 7x + 6x^3 + 3x^2 - 5 \]

  2. Combine like terms.

  \[ = 4x^3 + 6x^3 - 2x^2 + 3x^2 - 7x - 5 \]
  
  \[ = 10x^3 + x^2 - 7x - 5 \]
OPERATIONS ON POLYNOMIALS CONT’D

Techniques:

To multiply polynomials:

1. Multiply each term of one polynomial by each term of the other polynomial. (This is actually the distributive property applied more than once.)

2. Combine like terms.

Note: In the case of multiplying 2 binomials, the method is often referred to as FOIL for First, Outer, Inner, Last.

Examples:

Ex: Multiply \((2x^2 - 7x + 1)(3x + 4)\)

\[(2x^2 - 7x + 1)(3x + 4) = (2x^2)(3x)+2x^2(4)\]
\[-7x(3x)-7x(4)\]
\[+1(3x)+1(4)\]

\[= 6x^3+8x^2 -21x^2-28x +3x+4\]

Ex: Multiply \((x + 3)(x - 2)\)

\[(x + 3)(x - 2) = x^2 - 2x + 3x - 6\]

F O I L

\[= x^2 + x - 6\]

To divide a polynomial by a monomial, divide the monomial into each term of the polynomial.

Ex: Divide \(\frac{3x^2 + 6x + 10}{2x}\)

\[\frac{3x^2 + 6x + 10}{2x} = \frac{3x^2}{2x} + \frac{6x}{2x} + \frac{10}{2x}\]

\[= \frac{3x}{2} + 3 + \frac{5}{x}\]

To divide a polynomial, use the long division pattern for dividing whole numbers:

1. Arrange both polynomials in standard form with exponents in descending order. If either polynomial has a “missing term,” use zero as a placeholder.

2. Divide the first term of expression by 1st term of divisor.

3. Multiply the result by each term.

4. Subtract by changing each sign and combining like terms.

5. Bring down the next term in the dividend.

6. Repeat steps 1-4 until the process is complete.

7. Add to the resulting quotient the remainder divided by the divisor.
# FACTORING

**Techniques:**

<table>
<thead>
<tr>
<th>To factor a polynomial:</th>
<th>Examples:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Factor using the greatest common factor (GCF). (Divide each term by the largest expression that will divide into every term.)</td>
<td>Ex: Factor $8x^3 - 50x$ (2 terms) $8x^3 - 50x = 2x(4x^2 - 25)$ GCF</td>
</tr>
<tr>
<td>2. Use the factoring technique that corresponds to the number of terms.</td>
<td></td>
</tr>
<tr>
<td>(a) If there are 2 terms, use the difference of squares or the difference or sum of cubes formula. $a^2 - b^2 = (a + b)(a - b)$ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$</td>
<td>Ex: Factor $3x^3 - 24$ (2 terms) $3x^3 - 24 = 3(x^3 - 8)$ GCF $= 3(x - 2)(x^2 + 2x + 4)$ difference of cubes</td>
</tr>
<tr>
<td>(b) If there are 3 terms, use the reverse of FOIL trial-and-error technique.</td>
<td>Ex: Factor $5x^2 - 15x + 10$ (3 terms) $5x^2 - 15x + 10 = 5(x^2 - 3x + 2)$ GCF $= 5(x - 2)(x - 1)$ reverse of FOIL</td>
</tr>
<tr>
<td>(c) If there are 4 terms, use the grouping technique. (Factor the GCF out of the first 2 terms, then out of the second 2 terms, then out of the resulting final expression.</td>
<td>Ex: Factor $x^3 - 2x^2 + 4x - 8$ (4 terms) $x^3 - 2x^2 + 4x - 8 = x^2(x - 2) + 4(x - 2)$ Grouping $= (x - 2)(x^2 + 4)$</td>
</tr>
<tr>
<td>3. Check each step of the factoring with multiplication.</td>
<td>Check $F O I L$</td>
</tr>
</tbody>
</table>

Check $F O I L$ $= x^3 + 4x - 2x^2 - 8$ $\checkmark$
### RATIONAL EXPRESSIONS

#### Techniques:

**To simplify a rational expression**, completely factor the numerator and denominator and then cancel common factors.

#### Examples:

Ex: Simplify

\[
\frac{x^2 + 2x - 15}{3x - 9}
\]

\[
x^2 + 2x - 15 = (x + 5)(x - 3) = x + 5
\]

\[
3x - 9 = 3(x - 3)
\]

Ex: Simplify

\[
\frac{3}{x^2 - 9}
\]

\[
3(x - 3)
\]

#### To multiply or divide rational expressions:

1. Completely factor the numerator and denominator.

2. Perform the indicated operation.

3. Cancel common factors.

Ex: Divide:

\[
\frac{2x}{3x - 12} \div \frac{x^2 - 2x}{x^2 - 6x + 8}
\]

\[
\frac{2x}{3(x - 4)} \times \frac{(x - 4)(x - 2)}{x(x - 2)} = \frac{2}{3}
\]

Ex: Multiply:

\[
\frac{x}{5x^2 - 20x} \times \frac{x - 4}{2x^2 + x - 3}
\]

\[
\frac{x}{5x(x - 4)} \times \frac{x - 4}{(2x + 3)(x - 1)} = \frac{1}{5(2x + 3)(x - 1)}
\]

#### To add or subtract rational expressions:

1. Completely factor the numerator and denominator.

2. Find the least common denominator by using each factor represented, raised to the highest power occurring in each denominator.

3. Multiply numerator and denominator by an expression resulting in the common denominator.

4. Perform the operation and simplify.

Ex: Subtract

\[
\frac{6x}{x^2 - 4} - \frac{3}{(x - 2)^2}
\]

\[
= \frac{6x}{(x + 2)(x - 2)} - \frac{3}{(x - 2)^2}
\]

\[
= \frac{6x(x - 2)}{(x + 2)(x - 2)^2} - \frac{3(x + 2)}{(x + 2)(x - 2)^2}
\]

\[
= \frac{6x^2 - 12x - 3x + 6}{(x + 2)(x - 2)^2}
\]

\[
= \frac{6x^2 - 15x - 6}{(x + 2)(x - 2)^2}
\]
### GRAPHING

#### Points:

A point on the rectangular coordinate system can be represented by an ordered pair \((x, y)\). The first coordinate gives the position along the horizontal axis, and the second gives the vertical position.

**Examples:**

\[
\begin{align*}
\text{Ex: } & (0, -2) \text{ and } (-3, 2)
\end{align*}
\]

#### Intercepts:

To find the **x-intercept** of the graph of a given equation, let \(y = 0\) and solve for \(x\).

**Examples:**

\[
\begin{align*}
\text{x-intercept: } & 3x + 0 = 6 \\
& 3x = 6 \\
& x = 2 \\
& (0, 6)
\end{align*}
\]

To find the **y-intercept** of the graph of a given equation, let \(x = 0\) and solve for \(y\).

**Examples:**

\[
\begin{align*}
\text{y-intercept: } & 3(0) + y = 6 \\
& y = 6 \\
& (2, 0)
\end{align*}
\]

**Note:** A line may be graphed by finding, plotting, and connecting the intercepts.

#### Point Plotting:

The graph of an equation is the plotted set of all ordered pairs whose coordinates satisfy the equation.

**Ex:** Sketch the graph of

\[
\begin{align*}
& y - x^2 = 3 \\
& y = x^2 + 3
\end{align*}
\]

**Steps:**

1. Isolate one of the variables.

2. Make a table of values showing several solution points.

3. Plot the points in a rectangular coordinating system.

4. Connect these points with a smooth curve or line.
**GRAPHING CONT’D**

**Lines:**

The \textbf{y-intercept} is where the graph crosses the y-axis.

The \textbf{slope} measures the steepness of the graph and can be expressed as \( \frac{\text{rise}}{\text{run}} \).

Moving left to right on the graph, if the slope is \textbf{positive}, the graph rises.

If the slope is \textbf{negative}, the graph falls.

**Examples:**

Ex: Graph the line with y-intercept (0, 4) and slope \( m = -2 \).

The equation of this line is \( y = -2x + 4 \).

---

**To find the slope of the line through 2 points,**

use the following formula:

Given points \((x_1, y_1)\) and \((x_2, y_2)\)

\[
\text{slope } m = \frac{y_2 - y_1}{x_2 - x_1}
\]

Ex: Find the equation of the line through \((-3, 7)\) and \((3, 1)\)

\[
m = \frac{7 - 1}{-3 - 3} = \frac{6}{-6} = -1
\]

---

**To find the equation of the line with given characteristics,** use the following formula:

\( y - y_1 = m(x - x_1) \) is the line through the point \((x_1, y_1)\) with slope = \( m \)

Ex:

\[
y - 7 = -1(x + 3)
y = -x + 4
\]

---

The graph of \( y = mx + b \) is a \textbf{line} with slope = \( m \) and y-int = \((0, b)\).

Ex: \( y = -x + 4 \) is the equation of the line with slope = -1 and y-int = \((0, 4)\)

---

\( x = a \) is a \textbf{vertical line} through \((a, 0)\) with undefined slope.

\( y = b \) is a \textbf{horizontal line} through \((0, b)\) with slope zero.

Ex:

\( x = 2 \), slope is undefined
\[
y = 3, \text{ slope 0}
\]

---

**Parallel lines** have the same slopes.

**Perpendicular lines** have slopes that are negative reciprocals.

Ex: \( y = 2x + 5 \) and \( y = -\frac{1}{2}x + 3 \) are the equations of perpendicular lines since the slopes are 2 and \(-\frac{1}{2}\).
GRAPHING CONT’D

**Parabolas:**

The graph of \( f(x) = ax^2 + bx + c \) where \( a \neq 0 \) is a parabola with vertex \( \left( \frac{-b}{2a}, f\left( \frac{-b}{2a} \right) \right) \).

**If \( a \) is positive,** the parabola opens up, and a **minimum** occurs at the vertex.

**If \( a \) is negative,** the parabola opens downward, and a **maximum** occurs at the vertex.

**Examples:**

Ex: The graph of \( f(x) = x^2 + 2x + 4 \) is a parabola which opens upward that has a vertex \((-1, 3)\)

\[
x = \frac{-b}{2a} = \frac{-2}{2} = -1
\]

\[
y = f(-1) = (-1)^2 + 2(-1) + 4 = 1 - 2 + 4 = 3
\]

SIMPLIFYING RADICALS

**Techniques:**

To remove all possible factors from the radical:

1. Write the number as a product using the largest factor that is a perfect \( k \)th power where \( k \) is index.

2. If possible, write the variable factor as a product using the largest exponent that is a multiple of \( k \).

3. Apply the radical to each part of the fraction or product. The roots are written outside radical and the "leftover" factors remain under the radical.

**Examples:**

Ex: Simplify \( \sqrt{50} \)

\[
\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \sqrt{2} = 5\sqrt{2}
\]

Ex: Simplify \( \sqrt[3]{54x^6y^8} \)

\[
\sqrt[3]{54x^6y^8} = \sqrt[3]{27 \cdot 2x^6y^6y^2} = 3x^2y^2\sqrt[3]{2y^2}
\]
### SIMPLIFYING RADICALS CONT’D

**Techniques:**

| To rationalize a denominator with one term, multiply the numerator and denominator by a radical that will produce a perfect \( k \)th power radicand in the denominator and simplify. |
| Ex: Rationalize the denominator in \( \frac{1}{\sqrt[3]{x^2}} \) |
| \[
\frac{1}{\sqrt[3]{x^2}} = \frac{1}{\sqrt[3]{x^2}} \cdot \frac{\sqrt[3]{x}}{\sqrt[3]{x}} = \frac{\sqrt[3]{x}}{x}
\] |

| To rationalize a denominator with two terms, multiply the numerator and denominator by the conjugate of the denominator (opposite middle sign), then multiply using FOIL and simplify. |
| Ex: Rationalize the denominator in \( \frac{2}{3 - \sqrt{5}} \) |
| \[
\frac{2}{3 - \sqrt{5}} = \frac{2}{(3 - \sqrt{5})(3 + \sqrt{5})} = \frac{2(3 + \sqrt{5})}{9 - 5} = \frac{3 + \sqrt{5}}{2}
\] |

| To reduce the index of a radical, rewrite using a fractional exponent and reduce the fraction before converting back to radical notation. |
| Ex: Simplify: \( \sqrt[6]{x^2} \) |
| \[
\sqrt[6]{x^2} = x^{\frac{2}{6}} = x^{\frac{1}{3}} = \sqrt[3]{x}
\] |

**Note:** If the fraction cannot be reduced, try writing the radican using an exponent.

| Ex: Simplify: \( \sqrt[4]{9} \) |
| \[
\sqrt[4]{9} = 9^{\frac{1}{4}} = (3^2)^{\frac{1}{4}} = 3^{\frac{2}{4}} = 3^{\frac{1}{2}} = \sqrt{3}
\] |

**Note:** \( \sqrt{-1} = i \), an imaginary number.

| Ex: Simplify \( \sqrt{-8} \) |
| \[
\sqrt{-8} = \sqrt{-4 \cdot 2} = \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{2} = i \cdot 2 \cdot \sqrt{2} = 2i \sqrt{2}
\] |
OPERATIONS ON RADICALS

Techniques:  Examples:

A fractional exponent indicates that a radical should be applied to the base. The numerator of the exponent denotes the power to which the base is raised, and the denominator denotes the root to be taken.

Ex: \[8^{\frac{1}{2}} = \sqrt{8^2} = \sqrt{64} = 4\]

or

\[8^{\frac{1}{2}} = (\sqrt{8})^2 = 2^2 = 4\]

To add and subtract radicals (combine like radicals):

1. Simplify each radical.
2. Combine those having same index and radicand by adding/subtracting their coefficients.

Ex: \[\sqrt[3]{75} + \sqrt[3]{27} = \sqrt[3]{25 \cdot 3} + \sqrt[3]{9 \cdot 3} = 5\sqrt[3]{3} + 3\sqrt[3]{3} = 8\sqrt[3]{3}\]

To multiply radicals with the same indices, multiply the radicands.

Ex: \[\sqrt[3]{5} \cdot \sqrt[2]{2} = \sqrt[6]{5 \cdot 2} = \sqrt[6]{10}\]

To divide radicals with the same indices, divide the radicands.

Ex: \[\frac{\sqrt{27}}{\sqrt{3}} = \sqrt[6]{\frac{27}{3}} = \sqrt[6]{9} = 3\]

To multiply and divide radicals with different indices:

1. Write each radical with fractional exponents.
2. Rewrite each with a common denominator.
3. Convert to the radical form.
4. Multiply or divide as usual.

Ex: Multiply \(\sqrt[3]{x} \cdot \sqrt[3]{x}\)

\[\sqrt[3]{x} \cdot \sqrt[3]{x} = x^{\frac{1}{3}} \cdot x^{\frac{1}{3}} = x^{\frac{2}{3}} \cdot x^{\frac{2}{3}} = x^{\frac{5}{6}} = \sqrt[6]{x^5}\]

or

\[\sqrt[3]{x} \cdot \sqrt[3]{x} = x^{\frac{1}{3}} \cdot x^{\frac{1}{3}} = x^{\frac{3}{6}} \cdot x^{\frac{2}{6}} = \frac{6}{\sqrt[6]{x^5}} \cdot \frac{6}{\sqrt[6]{x^2}} = 6 \sqrt[6]{x^3} \cdot \sqrt[6]{x^2} = 6 \sqrt[6]{x^5}\]
# SPECIAL EQUATIONS

## Techniques:

### Quadratic Equation
- **Quadratic Equation** – (2\(^\text{nd}\) degree equation) - equation that can be written in the form 
  \(ax^2 + bx + c = 0\), where \(a\), \(b\), and \(c\) are real and \(a \neq 0\).

## Examples:

### To solve:

1. If the equation has the form \(ax^2 + c = 0\) (no \(x\) term) isolate the squared quantity and extract square roots. **Don’t forget the ±.**
   
   **Ex:** Solve \(x^2 - 5 = 0\)  
   \[x^2 - 5 = 0\]  
   \[x^2 = 5\]  
   \[x = \pm \sqrt{5}\]

2. If zero is isolated and the expression \(ax^2 + bx + c\) will factor, then factor, set each factor equal to zero, and solve each equation.
   
   **Ex:** Solve \(2x^2 + 9x = 5\)
   \[2x^2 + 9x - 5 = 0\]
   \[(2x - 1)(x + 5) = 0\]
   \[2x - 1 = 0 \quad x + 5 = 0\]
   \[x = \frac{1}{2} \quad x = -5\]

3. The **Quadratic Formula** can be used on any quadratic equation. Set one side equal to zero, identify \(a\), \(b\), and \(c\) and substitute into formula.
   
   If \(ax^2 + bx + c = 0\), then
   
   \[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

   **Ex:** Solve \(x^2 + 7x + 4 = 0\)
   \(x^2 + 7x + 4 = 0\) will not factor  
   \(a=1, \ b=7, \ c=4\)
   
   \[x = \frac{-7 \pm \sqrt{49 - 4(1)(4)}}{2(1)}\]
   
   \[x = \frac{-7 \pm \sqrt{33}}{2}\]
### Radical Equation – equation in which the variable occurs in a radical or is raised to a fractional exponent.

#### To solve:
1. Isolate the most complicated radical on one side.
2. Raise each side to the power equal to the index of the radical.
3. If the radical remains, repeat steps 1 and 2.
4. Solve the resulting equation.
5. A check is necessary if the original equation involves a radical with an even index.

**Example:**

\[
\sqrt{x + 2} + 4 = 10
\]

1. \(\sqrt{x + 2} + 4 = 10\)
2. \(\sqrt{x + 2} = 6\)
3. \((\sqrt{x + 2})^2 = 6^2\)
4. \(x + 2 = 36\)
5. \(x = 34\)

**Check:**

\[
\sqrt{34 + 2} + 4 = 10
\]

\[
\sqrt{36 + 4} = 10
\]

\[
x + 4 = 10
\]

\[
10 = 10 \checkmark
\]

### Higher - Order Factorable Equation - equation in which zero is isolated and the polynomial on the other side is factorable.

#### To solve:
1. Isolate zero.
2. Factor.
3. Set each factor equal to zero and solve each equation.

**Example:**

\[
x^3 = 4x
\]

1. \(x^3 = 4x\)
2. \(x(x^2 - 4) = 0\)
3. \(x(x + 2)(x - 2) = 0\)

\[
x = 0 \quad x + 2 = 0 \quad x - 2 = 0
\]

\[
x = 0 \text{ or } x = -2 \text{ or } x = 2
\]
MORE SAMPLE QUESTIONS:

More sample questions, including math questions, are available at the following link:


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