NC DAP

(North Carolina Diagnostic Assessment and Placement)

Mathematics

Study Guide

(Revised 11.18.15)

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NC DAP

MATHEMATICS
STUDY GUIDE

All students are encouraged to prepare for placement testing. Not reviewing for the placement tests could result in students being placed into courses below their actual skill level. This can delay student progress, so prepare as best you can!

The placement test will determine which English and math courses you will take when you attend Coastal. All of our courses are designed to help you succeed, so you will be in good hands, no matter where you place.

This study guide contains reminders, testing tips, an overview of the test, and sample questions.

The faculty and staff of Coastal Carolina Community College wish you the best of luck as you embark on your educational path. We look forward to working with you!

Reminders

- The test is computerized; you will be furnished scrap paper and pencil to make notes and/or calculations.
- Bring a photo ID and your student ID number on test day.
- With the exception of the essay (2 hour limit), each section is untimed.
- Once an answer is submitted, it cannot be changed.
- Unauthorized devices such as cell phones and iPads are not allowed.
- Work by yourself.

Testing Tips

- Get plenty of rest the night before you plan to take the test.
- Make sure you eat a good breakfast or lunch prior to testing.
- Take the test seriously; you may only test twice in a 12 month period.
- Don't be discouraged; this test is designed to feel difficult.
- Write as much as you can for the essay; don't just stop when you reach the required word count. Whatever you do, don't skip this section!

Mathematics Overview
The NCCCS Diagnostic and Placement Mathematics test contains 72 questions that measure proficiency in six content areas. *This test is untimed.* The six content areas are as follows:

Operations with Integers — Topics covered in this category include:
- Problem events that require the use of integers and integer operations
- Basic exponents, square roots and order of operations
- Perimeter and area of rectangles and triangles
- Angle facts and the Pythagorean Theorem

Fractions and Decimals — Topics covered in this category include:
- Relationships between fractions and decimals
- Problem events that result in the use of fractions and decimals to find a solution
- Operations with fractions and decimals
- Circumference and area of circles
- The concept of π
- Application problems involving decimals

Proportions, Ratios, Rates and Percentages — Topics covered in this category include:
- Conceptual application problems containing ratios, rates, proportions and percentages
- Applications using U.S. customary and metric units of measurement
- Geometry of similar triangles

Expressions, Linear Equations and Linear Inequalities — Topics covered in this category include:
- Graphical and algebraic representations of linear expressions, equations and inequalities
- Application problems using linear equations and inequalities

Graphs and Equations of Lines — Topics covered in this category include:
- Graphical and algebraic representations of lines
- Interpretation of basic graphs (line, bar, circle, etc.)

Polynomials and Quadratic Applications — Topics in this category include:
- Graphical and algebraic representations of quadratics
- Finding algebraic solutions to contextual quadratic applications
- Polynomial operations
- Factoring polynomials
- Applying factoring to solve polynomial equations

DEVELOPMENTAL MATH (DMA) CLASSES	STUDY/FOCUS AREAS
PAGE-BY-PAGE STUDY KEY	
DMA 010	Pages 10-14 and 16
DMA 020	Pages 5-8 and 16
DMA 030	Pages 8-10
DMA 040	Pages 15-19
DMA 050	Pages 25-26
DMA 060	Pages 20-23 and 31
DMA 070	Page 24
DMA 080	Pages 27-31

FRACTIONS

Method:

To simplify a fraction, divide the numerator and denominator by all common factors.

Ex: $\frac{12}{18} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}$

To multiply fractions:

1. Divide out all factors common to a numerator and any denominator.

Ex: $\frac{5}{6} \times \frac{4}{15} = \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}} \times \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{15}}} = \frac{2}{9}$

2. Multiply numerators.
3. Multiply denominators.

To divide fractions, invert the second fraction and multiply.

Ex: $\frac{5}{6} \div \frac{1}{2} = \frac{5}{6} \times \frac{2}{1} = \frac{5}{\underset{3}{\cancel{6}}} \times \frac{\overset{1}{\cancel{2}}}{1} = \frac{5}{3}$

To add or subtract fractions:

1. Find the Least Common Denominator (LCD)

Ex: $\frac{1}{2} + \frac{3}{7} + \frac{5}{8}$ **LCD=56**

2. In each fraction, multiply the numerator and denominator by the same number to obtain the common denominator.

$\frac{1^{\times 28}}{2_{\times 28}} + \frac{3^{\times 8}}{7_{\times 8}} + \frac{5^{\times 7}}{8_{\times 7}}$

3. Add or subtract the numerators and keep the common denominator.

$\frac{28}{56} + \frac{24}{56} + \frac{35}{56} = \frac{87}{56}$

To change a mixed number to an improper fraction:

1. Multiply to the denominator by the whole number.

Ex: $5\tfrac{2}{3} = \frac{3 \times 5 + 2}{3} = \frac{17}{3}$

2. Add the product to the numerator.
3. Place the sum over the denominator.

To change an improper fraction to a mixed number:

1. Divide the denominator into the numerator. **Ex:** $\frac{42}{5} = 8\frac{2}{5}$ since $5\overline{)42}$ gives quotient 8: $42 - 40 = 2$

2. The whole number in the mixed number is the quotient, and the fraction is the remainder over the denominator.

To change a whole number to a fraction, write the number over 1. **Ex:** $9 = \frac{9}{1}$

To multiply or divide the whole numbers and/or mixed numbers:

1. Change to improper fractions. **Ex:** $2\frac{1}{3} \div 5$

2. Multiply or divide the fractions $= \frac{7}{3} \div \frac{5}{1} = \frac{7}{3} \times \frac{1}{5} = \frac{7}{15}$

3. Change the answer to mixed number.

To add mixed numbers, change to improper fractions, add, and then convert to a mixed number **Ex:** $6\frac{3}{4} + 2\frac{5}{8}$

$$= \frac{27}{4} + \frac{21}{8} = \frac{54}{8} + \frac{21}{8} = \frac{75}{8} = 9\frac{3}{8}$$

OR

Add the whole numbers and fractions separately. **Ex:**

$$\begin{array}{r} 6\frac{3}{4} \\ +\,2\frac{5}{8} \\ \hline \end{array} \quad \begin{array}{r} 6\frac{6}{8} \\ +\,2\frac{5}{8} \\ \hline 8\frac{11}{8} \end{array} \quad = 8 + 1\frac{3}{8} = 9\frac{3}{8}$$

If an improper fraction results, change it to a mixed number and add the whole numbers.

To subtract mixed numbers, change to improper fractions, subtract, and convert to mixed numbers. **Ex:** $8\frac{1}{5} - 4\frac{2}{3}$

$$\frac{41}{5} - \frac{14}{3} = \frac{123}{15} - \frac{70}{15} = \frac{53}{15} = 3\frac{8}{15}$$

OR

Subtract the whole numbers and fractions separately. **Ex:**

$$\begin{array}{r} 8\frac{1}{5} \\ -\,4\frac{2}{3} \\ \hline \end{array} \quad \begin{array}{r} 8\frac{3}{15} \\ -\,4\frac{2}{3} \\ \hline \end{array} \quad \begin{array}{r} {}^{7}\not{8}\,\frac{15}{15}\frac{3}{15} \\ -\,4\frac{10}{15} \\ \hline \end{array} \quad \begin{array}{r} 7\frac{18}{15} \\ -\,4\frac{10}{15} \\ \hline 3\frac{8}{15} \end{array}$$

If necessary, borrow a fraction equal to 1 from the whole number.

DECIMALS

Method:

To determine which of two decimals is larger:

1. Write the decimals so that they have the same number of digits (by adding zeros).

2. Start at the left and compare corresponding digits. The larger number will have the larger digit.

Ex: .257 < .31

since .257 < .310

and 2 < 3

To round a decimal:

1. Locate the place for which the round off is required.

2. Compare the first digit to the right of this place to 5. If this digit is less than 5, drop it and all digits to the right of it. If this digit is greater than 5, increase the rounded digit by one and drop all digits to the right.

Ex: Round: **1.5725** to the
a) nearest hundredth
b) nearest thousandth

a) 1.5725 = 1.57
b) 1.5725 = 1.573

To add or subtract decimals:

1. Write the numbers vertically and line up the decimal points. If needed, add zeros to right of decimal digits.

2. Add or subtract as with whole numbers.

3. Align the decimal point in the answer with the other decimal points.

Ex: Add:
3.65 + 12.2 + .51

```
  3.65
 12.20
+  .51
 16.36
```

To multiply decimals:

1. Multiply the numbers as whole numbers.

2. Determine the sum of decimal places in the 2 numbers.

3. Make sure the answer has the same number of decimal places as the sum from Step 2. (Insert zeros to the left if necessary.)

Ex: Multiply:
.0023 × .14

```
  .0023
×   .14     6 decimal places total
   092
    23
.000322     6 decimal places in result
```

Decimals cont'd

Method:

To divide decimals:

1. Make the divisor a whole number by moving the decimal point to the right. (Mark this position with a caret ^.)
2. Move the decimal in the dividend to the right the same number of places. (Mark this position with a caret ^.)
3. Place the decimal point in the answer directly above the caret.
4. Divide as with whole numbers, adding zeros to the right if necessary. Continue until the remainder is zero, the decimal digits repeat, or the desired number of decimal positions is achieved.

Examples

Ex: Divide: **0.168 by 0.05**

```
          3.36
.05^ )0.16^80
        15
         18
         15
          30
          30
           0
```

To convert a fraction to a decimal, divide the denominator into the numerator.

Ex: Convert: $9/11$ to a decimal

$$11\overline{)9.0000} = .8181... = .\overline{81}$$

PERCENTS

Technique:

% means "per 100" or "out of 100"

To convert a percentage to a fraction or decimal, divide by 100%.

Note: A shortcut for dividing by 100 is moving the decimal 2 places to the left.

Examples

Ex: 45% means $\frac{45}{100}$ or 45 out of 100.

Ex: 100% is equal to $\frac{100}{100}$ or 1

Ex: Convert: **32% to a fraction**.

$$32\% = \frac{32\%}{100\%} = \frac{32}{100} = \frac{8}{25}$$

Ex: Convert: **2.5% to a decimal.**

$$2.5\% = \frac{2.5\%}{100\%} = \frac{2.5}{100} = .025$$

To convert a fraction or decimal to a percentage, multiply by 100%

Ex: Convert: $3/5$ **to a percent.**

$$\frac{3}{5}=\frac{3}{5}\times 100\% = \frac{3}{{}_{1}\not{5}}\times\frac{\not{100}^{20}}{1}\% = 60\%$$

Note: A shortcut for multiplying by 100 is moving the decimal 2 places to the right.

Ex: Convert: 1.4 to a percent

$1.4 \times 100\% = 140\%$

To solve percent equations:

1. Change the percent to a decimal or fraction.
2. Translate the question to an equation by replacing "is" with =, "of" with multiplication, and "what" with a variable.
3. Solve the equation for the variable.

Ex: 14 is 25% of what number?

$14 = .25x$

$$\frac{14}{.25} = \frac{.25x}{.25}$$

$56 = x$

so 14 is 25% of 56

To solve a percent increase or decrease problem, use the following models:

Increase

new = original + percent as decimal × original

Ex: If 68,000 was increased to 78,500, find the percent increase.

$78{,}500 = 68{,}000 + x(68{,}000)$

$10{,}500 = 68{,}000x$

$$\frac{10{,}500}{68{,}000} = \frac{68{,}000x}{68{,}000}$$

$.154 = x$ So the percent increase is 15.4%

Decrease

new = original - percent as decimal × original

VARIOUS APPLICATIONS

Technique:

Examples

To convert units, multiply by a fraction consisting of a quantity divided by the same quantity with different units (multiplication by 1). Set up the fraction so that the units will cancel appropriately.

Ex: Convert 5 meters to feet using the fact that 1 ft = .305 meters

1ft = .305 meters

$$5 \text{ meters} = 5\ \cancel{\text{meters}}\left(\frac{1\text{ft}}{.305\cancel{\text{meters}}}\right) = \frac{5\text{ft}}{.305} = 16.39\text{ft}$$

To solve problems involving vehicle travel, use Distance = Rate × Time

Ex: How long does it take a car traveling 55 mph to travel 30 miles?

$d = rt$

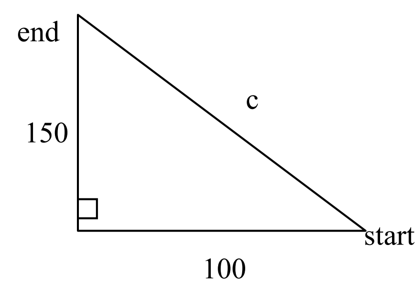
$30 = 55t$

$30/55 = t$

t = .55 hours or 33 minutes

To solve problems involving sides of a right triangle, use the Pythagorean Theorem. In a right triangle, if a and b are the legs and c is the hypotenuse, $a^2 + b^2 = c^2$.

Ex: A plane flew in a straight line to a point 100 miles west and 150 miles north from where it began. How far did that plane travel?



$c^2 = 100^2 + 150^2$

$c^2 = 32500$

$c = \sqrt{32500}$

$c = 180.28$ miles

The plane flew 180.28 miles

TRANSLATING PHRASES TO ALGEBRAIC EXPRESSIONS

Verbal Description	Algebraic Operation or Symbol	Examples
is, equal, are, results in	equal sign	**Ex:** a number plus 7 results in 10 $x + 7 = 10$
sum, plus, increased, by, greater than, more than, exceeds, total of	addition	**Ex:** the sum of a number and 2 $x + 2$
difference, minus, decreased by, less than, subtracted from, reduced by, the remainder	subtraction	**Ex:** 7 subtracted from 5 $5 - 7$ **Ex:** 3 less than a number $x - 3$
product, multiplied by, twice, times, of	multiplication	**Ex:** twice a number $2x$
quotient, divided by, ratio, per	division	**Ex:** 35 miles per hour $\frac{35\ miles}{1\ hour}$
exponent, power, squared, cubed	exponent	**Ex:** two cubed 2^3 **Ex:** a number to the 5th power x^5

Note: ***Parentheses must be used to indicate an operation is to be applied to an entire expression.***

Ex: five times the sum of a number and 3
$5(x + 3)$

Note: ***Use a variable to represent an unknown quantity.***

SETS OF REAL NUMBERS

Definitions:

Examples

Integers = all positive and negative whole numbers and zero.

Ex: -100, 20, 0, -451 (all integers)

Rational Numbers = all terminating or repeating decimals.

Ex: $\frac{1}{4} = .25$ and $\frac{2}{3} = \overline{.6}$ are rational

Irrational Number = all nonterminating, nonrepeating decimals.

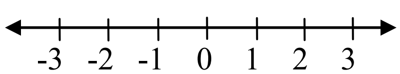
Ex: $\sqrt{2} = 1.4142135\ldots$ is irrational.

Ex: $\pi = 3.1415926\ldots..$ is irrational.

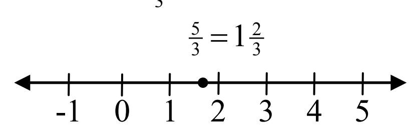
Prime Number = positive integer greater than 1 with no integer factors other than itself and 1.

Ex: 5, 17, 29, and 37 (all prime)

Real Number Line



Ex: Plot $\frac{5}{3}$ on the number line.



Absolute Value

$|a|$ = the distance between a and 0 on the number line.

To determine absolute value, just make the number positive

Ex: 2 = 2

0 = 0

-3 = 3

OPERATIONS ON INTEGERS

Techniques:

Examples

Addition

If the numbers have the same signs, add the absolute value and attach the common sign to the result.

Ex: $5 + 12 = 17$
Ex: $-4 + (-10) = -14$
Ex: $(-3) + (-7) + (-10) = -20$

If the numbers have opposite signs, subtract the smaller absolute value from the larger and attach the sign of the larger.

Ex: $5 + (-12) = -7$
Ex: $(-7) + 3 = -4$
Ex: $-2 + 10 = 8$

Subtraction

Add the opposite of the second number.
Subtracting a positive is the same as adding a negative.
Subtracting a negative is the same as adding a positive.

Ex: $7 - 15 = 7 + (-15) = -8$
Ex: $4 - 5 = 4 + (-5) = -1$
Ex: $7 - (-3) = 7 + 3 = 10$

Multiplication and Division

If the numbers have the same signs, perform the operation on the absolute values and attach a positive sign to the result.

Ex: $(-6) \div (-2) = 3$
Ex: $\frac{-12}{-4} = 3$
Ex: $(-3)(-2) = 6$

If the numbers have opposite signs, perform the operation on the absolute values and attach a negative sign to the result.

Ex: $(-5)(2) = -10$
Ex: $\frac{-15}{3} = -5$

Exponents (positive integer exponents)

Multiply the base the number of times given by the exponent.

Ex: $(-2)^3 = (-2)(-2)(-2) = -8$

Ex: $2^5 = (2)(2)(2)(2)(2) = 32$

Ex: $-2^4 = -1(2)(2)(2)(2) = -16$

Ex: $(-2)^4 = (-2)(-2)(-2)(-2) = 16$

ORDER OF OPERATIONS

Techniques:

To evaluate an expression using the order of operations, perform operations in the following order:

1. **Parentheses**
   Starting with the innermost symbol, perform operations inside symbols of grouping (parentheses or brackets) or absolute value symbols.

2. **Exponents**
   Evaluate all exponential expressions.

3. **Multiplication/Division**
   In order from left to right, perform all multiplications and divisions

4. **Addition/Subtraction**
   In order from left to right, perform all additions and subtractions.

Examples

Ex: **Evaluate** $16 \div 2^3 - 4(3 - |5 - 7|) + 5$

$= 16 \div 2^3 - 4(3 - |5-7|) + 5$

$= 16 \div 2^3 - 4(3 - |-2|) + 5$

$= 16 \div 2^3 - 4(3 - 2) + 5$

$= 16 \div 2^3 - 4(1) + 5$

$= 16 \div 8 - 4(1) + 5$

$= 2 - 4(1) + 5$

$= 2 - 4 + 5$

$= -2 + 5$

$= 3$

EVALUATING ALGEBRAIC EXPRESSIONS

Techniques:

To evaluate an expression at given values of the variables:

1. Replace every occurrence of each variable with the appropriate real number. Use parentheses when substituting a negative number for a variable.

2. Use the order of operations to evaluate the resulting expression.

Examples

Ex: **Evaluate $y^2 - x + 2z + z^3$** when: $x = -3$, $y = -5$ and $z = 2$

$$y^2 - x + 2z + z^3$$
$$= (-5)^2 - (-3) + 2(2) + 2^3$$
$$= 25 + 3 + 4 + 8$$
$$= 28 + 12$$
$$= 40$$

SIMPLIFYING ALGEBRAIC EXPRESSIONS

Techniques:

To simplify an algebraic expression:

1. Starting with the innermost set, remove symbols of grouping. Usually the distributive property and/or rules of exponents must be used in this step.

2. Combine like terms by adding the coefficients of terms having the same variable factor.

Examples

Ex: Simplify: $\mathbf{(5x)^2 + 4[x^2 - (2x - 5)]}$

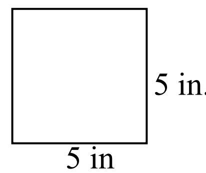
$$= (5x)^2 + 4[x^2 - 2x + 5]$$
$$= 25x^2 + 4x^2 - 8x + 20$$
$$= 29x^2 - 8x + 20$$

PERIMETER, AREA, AND VOLUME

Formulas: **Examples**

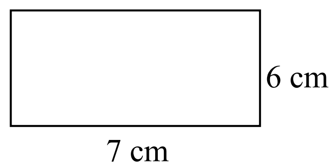
The following notation is used in the formulas:
l = length, w = width, h = height,
b = base, r = radius, a = area, v = volume
c = circumference, p = perimeter

Square $a = w^2$



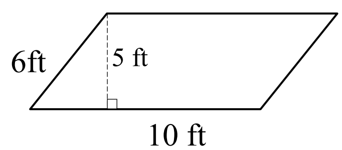
$a = 25in^2$

Rectangle $a = lw$



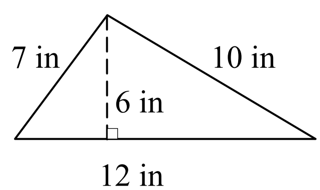
$a = 42cm^2$

Parallelogram $a = bh$



$a = 10(5) = 50ft^2$

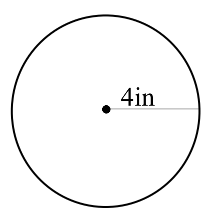
Triangle $a = \frac{1}{2} bh$



$a = \frac{1}{2}(12)(6) = 36in^2$

Circle $a = \pi r^2$ $\pi = 3.14$

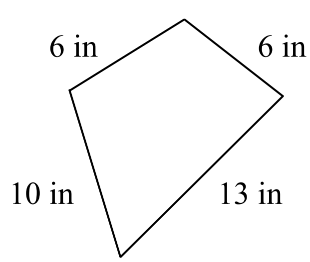
$c = 2\pi r$



$a = \pi(4)^2 = 50.27in^2$

$c = 2\ \pi(4) = 25.13in$

Any Polygon P= sum of the sides

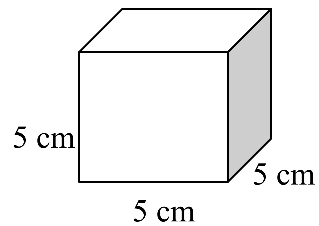


P = 6 + 6 + 13 + 10 = 35in

Perimeter, Area, and Volume cont'd

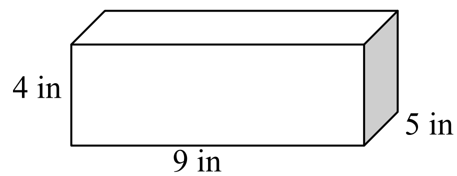
Formula | **Examples**

Cube $V = w^3$



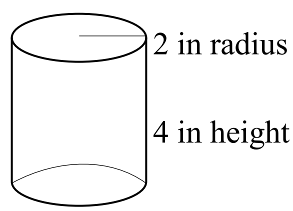
$V = 125cm^3$

Rectangular Solid $v = lwh$



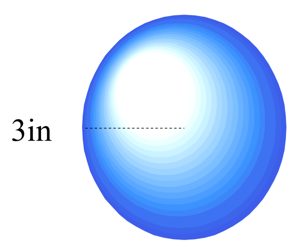
$V = 9 \times 4 \times 5$
$V = 180in^3$

Cylinder $v = \pi r^2 h$



$V = \pi\, 2^2 (4)$
$V \approx 50.27in^3$

Sphere $v = \frac{4}{3} \pi r^3$



$V = \frac{4}{3} \pi (3)^3$
$V \approx 113.1in^3$

LINEAR EQUATIONS

Techniques:

Examples

To solve a linear equation:

Ex: Solve $\frac{2}{3}(x + 2) + \frac{5}{6}x = x + 4$

1. Remove fractions by multiplying both sides by the lease common denominator (LCD).

LCD = 6 so multiply each term by 6 on both sides

$6 \cdot \frac{2}{3}(x+2) + 6 \cdot \frac{5}{6}x = 6(x+4)$

2. Simplify each side by distributing and combining like terms.

$4(x + 2) + 5x = 6(x + 4)$

$4x + 8 + 5x = 6x + 24$

3. Add or subtract variable terms from both sides so that the variable will occur on only one side.

$9x + 8 = 6x + 24$

$-6x \qquad -6x$

$3x + 8 = 0 + 24$

4. Isolate the variable by performing the same operation on both sides of the equation.

$3x + 8 = 24$

$-8 \quad -8$

$\frac{3x}{3} = \frac{16}{3}$

$x = \frac{16}{3}$

Note: A check is necessary if the equation contains fractions with variable denominators.

To solve an application problem using a linear equation.

Ex: *A person has 90 coins in quarters and dimes with a combined value of $16.80. Determine the number of coins of each type.*

1. Write a verbal equivalence containing the quantities involved in the problem.

Value of quarters + value of dimes = total value

2. Substitute given values for known quantities.

.25(number of quarters) + .10(number of dimes) = total value

.25(number of quarters) + .10(number of dimes) = $16.80

3. Use a variable to represent one unknown quantity in the equation.

.25x + .10(number of dimes) = 16.80

4. Replace the remaining unknown quantities with an appropriate expression involving the variable.

$.25x + .10(90 - x) = 16.80$

5. Solve the equation.

$.25x + 9 - .10x = 16.80$

$.15x + 9 = 16.80$

$15x = 7.8$

$x = 52$

6. Answer the original question.

$x = 52 \qquad 90 - x = 90 - 52 = 38$

There are 52 quarters and 38 dimes.

INEQUALITIES

Techniques:

Examples

To solve a linear inequality, employ the same procedure used for solving equations, but remember to reverse the order symbol when multiplying or dividing both sides by a negative number.

Ex: Solve **$-2x < 20$**

$$\frac{-2x}{-2} < \frac{20}{-2} \text{ (divided by negative)}$$

$x > -10$ (reverse direction of symbol)

To solve a compound inequality, isolate the variable in the center by performing the same operation on all three parts.

Ex: Solve **$-13 \leq 6x - 1 < 3$**

$$-13 \leq 6x - 1 < 3$$
$$+1 \qquad\quad +1 \quad +1$$

$$\frac{-12}{6} \leq \frac{6x}{6} < \frac{4}{6}$$

$$-2 \leq x < \frac{2}{3}$$

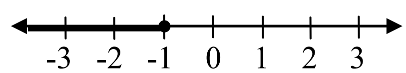
To graph an inequality:

1. Isolate the variable.
2. Shade in the numbers on the real number line that satisfy the inequality.
3. Use closed dots to show the endpoint is included with ≤ or ≥. Use open dot to show the endpoint is NOT included with < or >.

Ex: Graph **$x + 2 < 1$**

$$x + 2 < 1$$
$$-2 \quad -2$$

$$x < -1$$



To give the solution set of inequality in interval notation:

1. Isolate the variable.
2. List smallest number and largest number in the solution set separated with a comma. If no smallest number, use (-∞. If no largest number, use ∞)
3. Use brackets with ≤ or ≥.. Use parentheses with < or >.

Ex: Write **$-6 \leq x < 3$** in interval notation

[-6, 3)

Ex: Write **$x < 2$** in interval notation

(-∞, 2)

(-∞ is used since there is no smallest number that is less than 2.

EXPONENTS

Rules:

A **positive integer exponent** dictates the number of times the base is to be multiplied.

Ex: $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

Ex: $(-1)^5 = (-1)(-1)(-1)(-1)(-1)$
$= (1) \cdot (1) \cdot (-1)$
$= -1$

A **negative integer exponent** applied to a base is equal to the reciprocal of the base raised to the opposite exponent.

Ex: $2^{-3} = \frac{1}{2^3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8}$

Ex: $x^{-5} = \frac{1}{x^5}$

The **zero exponent** applied to any base **(except 0)** is equal to 1.

Ex: $4^0 = 1$

Ex: $(-12)^0 = 1$

Ex: $-(12)^0 = -1$

A factor can be moved from numerator to denominator (or vice versa) by changing the sign of the exponent.

Ex: $\frac{3^2 4^{-2}}{2^{-3}} = \frac{3^2 2^3}{4^2} = \frac{9 \cdot 8}{16} = \frac{9}{2}$

Ex: $\frac{3xy^{-3}}{z^{-4}} = \frac{3xz^4}{y^3}$

An exponent can be applied to each part of a **product** or **fraction**.

Ex: $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

Ex: $(3xy)^2 = 3^2x^2y^2 = 9x^2y^2$

To apply an exponent to a sum or difference, multiply the polynomial by itself.

Ex: $(x + 1)^2 = (x + 1)(x + 1)$
$= x^2 + 2x + 1$

OPERATIONS ON EXPONENTS

Procedures:

Examples

To multiply expressions with the same base, keep the base and add the exponents.

Ex: $x^2 \cdot x \cdot x^4 = x^{2+1+4} = x^7$

To divide expressions with the same base, keep the base and subtract the exponents.

Ex: $\dfrac{y^9}{y^5} = y^{9-5} = y^4$

Ex: $\dfrac{x^3}{x^7} = x^{3-7} = x^{-4} = \dfrac{1}{x^4}$

To raise an exponential expression to another power, keep the base and multiply the exponents.

Ex: $(x^3)^4 = x^{3(4)} = x^{12}$

To add or subtract expressions with exponents, combine like terms. The base and the exponent must be the same.

Ex: $3x^2 + 5x^2 = 8x^2$

Ex: $4x^2 - 3x - 6x^2$
$= 4x^2 - 6x^2 - 3x$
$= -2x^2 - 3x$

OPERATIONS ON POLYNOMIALS

Techniques:

Examples

To add polynomials, combine like terms (terms with the same variable raised to the same exponent.)

Ex: Simplify:
$\mathbf{(4x^3 - 2x^2 - 7x) + (-6x^3 - 3x^2 + 5)}$
$= 4x^3 - 2x^2 - 7x - 6x^3 - 3x^2 + 5$
$= 4x^3 - 6x^3 - 2x^2 - 3x^2 - 7x + 5$
$= -2x^3 - 5x^2 - 7x + 5$

To subtract polynomials:

1. Distribute the –1. (This will change the sign of each term on the 2nd polynomial.)
2. Combine like terms.

Ex: Simplify:
$\mathbf{(4x^3 - 2x^2 - 7x) - (-6x^3 - 3x^2 + 5)}$

$= 4x^3 - 2x^2 - 7x + 6x^3 + 3x^2 - 5$
$= 4x^3 + 6x^3 - 2x^2 + 3x^2 - 7x - 5$

$= 10x^3 + x^2 - 7x - 5$

To multiply polynomials:

1. Multiply each term of one polynomial by each term of the other polynomial. (This is actually the distributive property applied more than once.)
2. Combine like terms.

Ex: Multiply **$(2x^2 - 7x + 1)(3x + 4)$**

$(2x^2 - 7x + 1)(3x + 4)$

$= (2x^2)(3x)+2x^2(4)$

$-7x(3x)-7x(4)$

$+1(3x)+1(4)$

$= 6x^3+8x^2 -21x^2-28x +3x+4$

$= 6x^3 - 13x^2 - 25x + 4$

Note: In the case of multiplying 2 binomials, the method is often referred to as FOIL for First, Outer, Inner, Last.

Ex: Multiply **$(x + 3)(x - 2)$**

$(x + 3)(x - 2)$

$= x^2 - 2x + 3x - 6$

F O I L

$= x^2 + x - 6$

To divide a polynomial by a monomial, divide the monomial into each term of the polynomial.

Ex: Divide $\dfrac{3x^2 + 6x + 10}{2x}$

$$\frac{3x^2+6x+10}{2x} = \frac{3x^2}{2x} + \frac{6x}{2x} + \frac{10}{2x}$$

$$= \frac{3x}{2} + 3 + \frac{5}{x}$$

To divide a polynomial, use the long division pattern for dividing whole numbers:

1. Arrange both polynomials in standard form with exponents in descending order. If either polynomial has a "missing term," use zero as a placeholder.
2. Divide the first term of expression by 1st term of divisor.
3. Multiply the result by each term.
4. Subtract by changing each sign and combining like terms.
5. Bring down the next term in the dividend.
6. Repeat steps 1-4 until the process is complete.
7. Add to the resulting quotient the remainder divided by the divisor.

Ex: Divide $\dfrac{x^2 - 3}{x + 2}$

$$x - 2 + \frac{1}{x+2}$$

$$x + 2 \overline{) x^2 + 0x - 3}$$

$\underline{-x^2 \mp 2x}$

$-2x - 3$

$\underline{\pm 2x \pm 4}$

1

FACTORING

Techniques:

To factor a polynomial:

1. Factor using the greatest common factor (GCF). (Divide each term by the largest expression that will divide into every term.)
2. Use the factoring technique that corresponds to the number of terms.

   (a) If there are 2 terms, use the difference of squares or the difference or sum of cubes formula.

   $$\mathbf{a^2 - b^2 = (a + b)(a - b)}$$
   $$\mathbf{a^3 + b^3 = (a + b)(a^2 - ab + b^2)}$$
   $$\mathbf{a^3 - b^3 = (a - b)(a^2 + ab + b^2)}$$

   (b) If there are 3 terms, use the reverse of FOIL trial-and-error technique.

   (c) If there are 4 terms, use the grouping technique. (Factor the GCF out of the first 2 terms, then out of the second 2 terms, then out of the resulting final expression.

3. Check each step of the factoring with multiplication

Examples

Ex: Factor $\mathbf{8x^3 - 50x}$ **(2 terms)**

$8x^3 - 50x = 2x(4x^2 - 25)$ GCF

$= 2x(2x + 5)(2x - 5)$
difference of squares

Ex: Factor $\mathbf{3x^3 - 24}$ **(2 terms)**

$3x^3 - 24 = 3(x^3 - 8)$ GCF
$= 3(x - 2)(x^2 + 2x + 4)$
difference of cubes

Ex: Factor $\mathbf{5x^2 - 15x + 10}$ **(3 terms)**

$5x^2 - 15x + 10 = 5(x^2 - 3x + 2)$ GCF
$= 5(x - 2)(x - 1)$
reverse of FOIL

Ex: Factor $\mathbf{x^3 - 2x^2 + 4x - 8}$ **(4 terms)**

$x^3 - 2x^2 + 4x - 8$ Grouping
$= x^2(x - 2) + 4(x - 2)$
$= (x - 2)(x^2 + 4)$

Check
$(x - 2)(x^2 + 4)$
$= x^3 + 4x - 2x^2 - 8$ ✔
F O I L

RATIONAL EXPRESSIONS

Techniques:

To simplify a rational expression, completely factor the numerator and denominator and then cancel common factors.

Examples

Ex: Simplify:

$$\frac{x^2 + 2x - 15}{3x - 9}$$

$$\frac{x^2 + 2x - 15}{3x - 9} = \frac{(x + 5)\cancel{(x - 3)}}{3\cancel{(x - 3)}} = \frac{x + 5}{3}$$

To multiply or divide rational expressions:

1. Completely factor the numerator and denominator.
2. Perform the indicated operation.
3. Cancel common factors.

Ex: Divide:

$$\frac{2x}{3x - 12} \div \frac{x^2 - 2x}{x^2 - 6x + 8}$$

$$\frac{2x}{3(x - 4)} \times \frac{(x - 4)(x - 2)}{x(x - 2)}$$ factor, invert, and multiply

$$\frac{2\cancel{x}\cancel{(x - 4)}\cancel{(x - 2)}}{3\cancel{x}\cancel{(x - 4)}\cancel{(x - 2)}} = \frac{2}{3}$$

Ex: Multiply:

$$\frac{x}{5x^2 - 20x} \times \frac{x - 4}{2x^2 + x - 3}$$

$$\frac{x}{5x\cancel{(x - 4)}} \times \frac{\cancel{x - 4}}{(2x + 3)(x - 1)}$$

$$= \frac{1}{5(2x + 3)(x - 1)}$$

To add or subtract rational expressions:

1. Completely factor the numerator and denominator.
2. Find the least common denominator by using each factor represented, raised to the highest power occurring in each denominator.
3. Multiply numerator and denominator by an expression resulting in the common denominator.
4. Perform the operation and simplify.

Ex: Subtract: $\frac{6x}{x^2 - 4} - \frac{3}{(x - 2)^2}$

$$= \frac{6x}{(x + 2)(x - 2)} - \frac{3}{(x - 2)^2}$$ LCD = $(x + 2)(x - 2)^2$

$$= \frac{6x(x - 2)}{(x + 2)(x - 2)^2} - \frac{3(x + 2)}{(x + 2)(x - 2)^2}$$

$$= \frac{6x^2 - 12x}{(x + 2)(x - 2)^2} - \frac{3x + 6}{(x + 2)(x - 2)^2}$$

$$= \frac{6x^2 - 12x - 3x - 6}{(x + 2)(x - 2)^2}$$

$$= \frac{6x^2 - 15x - 6}{(x + 2)(x - 2)^2}$$

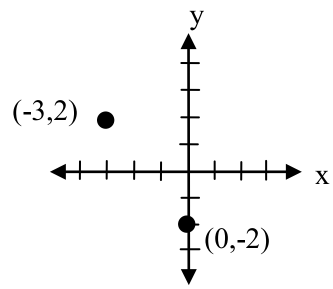
GRAPHING

Points:

A point on the rectangular coordinate system can be represented by an ordered pair (x,y). The first coordinate gives the position along the horizontal axis, and the second gives the vertical position.

Examples

Ex: (0, -2) and (-3, 2)



Intercepts:

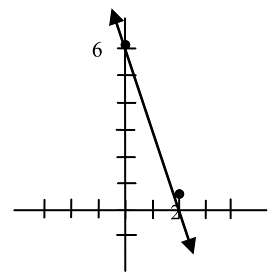
To find the x-intercept of the graph of a given equation, let y = 0 and solve for x.

To find the y-intercept of the graph of a given equation, let x = 0 and solve for y.

Note: A line may be graphed by finding, plotting, and connecting the intercepts.

Ex: Find the x and y of the graph of $3x + y = 6$

x-intercept:	y-intercept:
$3x + 0 = 6$	$3(0) + y = 6$
$3x = 6$	$y = 6$
$x = 2$	(0, 6)
(2, 0)	



Point Plotting:

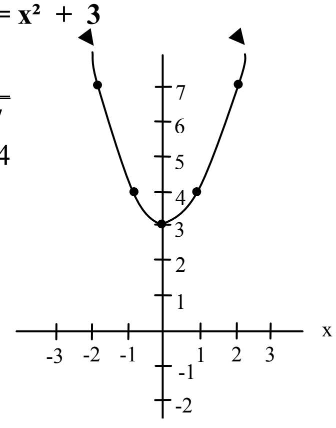
The graph of an equation is the plotted set of all ordered pairs whose coordinators satisfy the equation.

To sketch a graph using the point-plotting method:

1. Isolate one of the variables.
2. Make a table of values showing several solution points.
3. Plot the points in a rectangular coordinating system.
4. Connect these points with a smooth curve or line.

Ex: **Sketch: the graph of**
$\mathbf{y - x^2 = 3}$
$\mathbf{y = x^2 + 3}$

x	y
-2	$(-2)^2 + 3 = 7$
-1	$(-1)^2 + 3 = 4$
0	$0^2 + 3 = 3$
1	$1^2 + 3 = 4$
2	$2^2 + 3 = 7$



Lines:

To find the slope of the line through 2 points, use the following formula:
Given points (x_1, y_1) and (x_2, y_2)

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Ex: Find the equation of the line through (-3, 7) and (3, 1)

$$m = \frac{7-1}{-3-3} = \frac{6}{-6} = -1$$

To find the equation of the line with given characteristics, **use the following formula:**
$y - y_1 = m(x - x_1)$ is the line through the point (x_1, y_1) with slope = m

$y - y_1 = m(x - x_1)$
$y - 7 = -1(x + 3)$
$y - 7 = -x - 3$
$y = -x + 4$

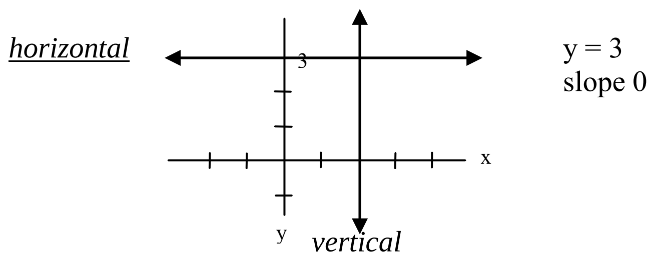
The graph of y = mx + b is a <u>line</u> with slope = m and y-int = (0, b).

Ex: y = -x + 4 is the equation of the line with slope = -1 and y-int = (0, 4)

x = a is a **<u>vertical line</u>** through (a, 0) with undefined slope.

y = b is a **<u>horizontal line</u>** through (0, b) with slope zero.

Ex: x = 2, slope is undefined



<u>Parallel lines</u> have the same slopes.

<u>Perpendicular lines</u> have slopes that are negative reciprocals.

Ex: y = 2x + 5 and y = ½x + 3 are the equations of <u>perpendicular lines</u> since the slopes are 2 and -½.

Parabolas:

The graph of $f(x) = ax^2 + bx + c$ where $a \neq 0$ is a parabola with vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

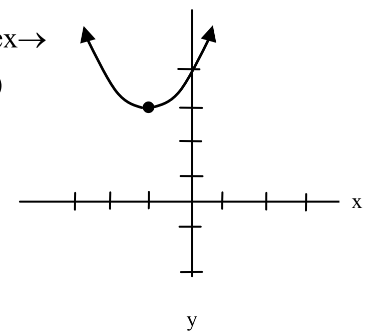
If a is positive, the parabola opens up.

If a is negative, the parabola opens downward.

Ex: The graph of $f(x) = x^2 + 2x + 4$ is a parabola which opens upward that has a vertex (-1, 3)

$$x = \frac{-b}{2a} = \frac{-2}{2} = -1$$

$$y = f(-1) = (-1)^2 + 2(-1) + 4$$
$$= 1 - 2 + 4 = 3$$



SIMPLIFYING RADICALS

Techniques:

To remove all possible factors from the radical:

1. Write the number as a product using the largest factor that is a perfect kth power where k is index.
2. If possible, write the variable factor as a product using the largest exponent that is a multiple of k.
3. Apply the radical to each part of the fraction or product. The roots are written outside radical and the "leftover" factors remain under the radical.

Examples

Ex: Simplify $\sqrt{50}$

$$\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25}\,\sqrt{2} = 5\sqrt{2}$$

Ex: Simplify $\sqrt[3]{54x^6y^8}$

$$\sqrt[3]{54x^6y^8} = \sqrt[3]{27 \cdot 2x^6y^6y^2}$$

$$= 3x^2y^2\sqrt[3]{2y^2}$$

Simplifying Radicals cont'd

To rationalize a denominator with one term,

multiply the numerator and denominator by a

radical that will produce a perfect kth power radicand in the denominator and simplify.

Ex: Rationalize the denominator in $\frac{1}{\sqrt[3]{x^2}}$

$$\frac{1}{\sqrt[3]{x^2}} = \frac{1}{\sqrt[3]{x^2}} \cdot \frac{\sqrt[3]{x}}{\sqrt[3]{x}} = \frac{\sqrt[3]{x}}{\sqrt[3]{x^3}} = \frac{\sqrt[3]{x}}{x}$$

To rationalize a denominator with

two terms, multiply the numerator

and denominator by the conjugate of

denominator (opposite middle sign),
then multiply using FOIL and simplify.

Ex: Rationalize the denominator in $\frac{2}{3-\sqrt{5}}$

$$\frac{2}{3-\sqrt{5}} = \frac{2}{(3-\sqrt{5})} \cdot \frac{(3+\sqrt{5})}{(3+\sqrt{5})} = \frac{2(3+\sqrt{5})}{9-5} = \frac{2\left(3+\sqrt{5}\right)}{4}$$

<u>**Techniques:**</u>

<u>**Examples**</u>

To reduce the index of a radical,

rewrite using a fractional exponent
and reduce the fraction before
converting back to radical notation.

Ex: Simplify: $\sqrt[6]{x^2}$

$$\sqrt[6]{x^2} = x^{\frac{2}{6}} = x^{\frac{1}{3}} = \sqrt[3]{x}$$

Note: If the fraction can not be reduced, try writing the radican using an exponent.

Ex: Simplify: $\sqrt[4]{\mathbf{9}}$

$$\sqrt[4]{9} = 9^{\frac{1}{4}} = \left(3^2\right)^{\frac{1}{4}} = 3^{\frac{2}{4}} = 3^{\frac{1}{2}} = \sqrt{3}$$

Note: $\sqrt{-1} = i$ ***, an imaginary number.***

Ex: Simplify $\sqrt{-\mathbf{8}}$

$$\sqrt{-8} = \sqrt{-4 \cdot 2}$$
$$= \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{2}$$
$$= i\,2\sqrt{2}$$
$$= 2i\sqrt{2}$$

OPERATIONS ON RADICALS

Techniques: | **Examples**

A **fractional exponent** indicates that a **radical** should be applied to the base. The **numerator** of the exponent denotes the **power** to which the base is raised, and the **denominator** denotes the **root** to be taken.

Ex: $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$

or

$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

To add and subtract radicals (combine like radicals):

Ex: **Add:** $\sqrt{75} + \sqrt{27}$

1. Simplify each radical.
2. Combine those having same index and radicand by adding/subtracting their coefficients.

$\sqrt{75} + \sqrt{27} = \sqrt{25 \cdot 3} + \sqrt{9 \cdot 3}$

$= 5\sqrt{3} + 3\sqrt{3} = 8\sqrt{3}$

To multiply radicals with the same indices, multiply the radicands.

Ex: $\sqrt{5} \cdot \sqrt{2} = \sqrt{5(2)} = \sqrt{10}$

To divide radicals with the same indices, divide the radicands.

Ex: $\dfrac{\sqrt{27}}{\sqrt{3}} = \sqrt{\dfrac{27}{3}} = \sqrt{9} = 3$

To multiply and divide radicals with different indices:

Ex: Multiply $\sqrt{x}\ \sqrt[3]{x}$

1. Write each radical with fractional exponents.
2. Rewrite each with a common denominator.
3. Convert to the radical form.
4. Multiply or divide as usual.

$\sqrt{x}\ \sqrt[3]{x} = x^{\frac{1}{2}} x^{\frac{1}{3}}$

$= x^{\frac{3}{6}} x^{\frac{2}{6}}$

$= x^{\frac{5}{6}} = \sqrt[6]{x^5}$

OR

$\sqrt{x}\ \sqrt[3]{x} = x^{\frac{1}{2}} x^{\frac{1}{3}}$

$= x^{\frac{3}{6}} x^{\frac{2}{6}}$

$= \sqrt[6]{x^3}\ \sqrt[6]{x^2}$

$= \sqrt[6]{x^3 \cdot x^2}$

$= \sqrt[6]{x^5}$

SPECIAL EQUATIONS

Techniques: | **Examples**

Quadratic Equation – (2^{nd} degree equation) -
equation that can be written in the form $ax^2 + bx + c = 0$ where a, b, and c are real and $a \neq 0$.

To solve:

1. If the equation has the form $ax^2 + c =$ (no x term) isolate the squared quantity and extract square roots. *Don't forget the* ±.

Ex: Solve $\mathbf{x^2 - 5 = 0}$

$x^2 - 5 = 0$ has no x-term

$x^2 = 5$

$x = \pm\sqrt{5}$

2. If zero is isolated and the expression $ax^2 + bx + c$ will factor, then factor, set each factor equal to zero, and solve each equation.

Ex: Solve $\mathbf{2x^2 + 9x = 5}$

$2x^2 + 9x - 5 = 0$

$(2x - 1)(x + 5) = 0$

$2x - 1 = 0$ $x + 5 = 0$

$x = ½$ $x = -5$

3. The Quadratic Formula can be used on any quadratic equation. Set one side equal to zero, identify a, b, and c and substitute into formula.

If $ax^2 + bx + c = 0$, Then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex: Solve $\mathbf{x^2 + 7x + 4 = 0}$

$x^2 + 7x + 4 = 0$ will not factor

a=1, b=7, c=4

$$x = \frac{-7 \pm \sqrt{49 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{33}}{2}$$

Special Equations cont'd

Techniques:

Radical Equation – equation in which the variable occurs in a radical or is raised to a fractional exponent.

To solve:
1. Isolate the most complicated radical on one side.
2. Raise each side to the power equal to the index of the radical.
3. If the radical remains, repeat steps 1 and 2.
4. Solve the resulting equation.
5. A check is necessary if the original equation involves a radical with an even index.

Examples

Ex: Solve $\sqrt{x+2}+4=10$

$\sqrt{x+2}+4=10$

$\sqrt{x+2}=6$

$(\sqrt{x+2})^2=6^2$

$x + 2 = 36$

$x = 34$

Check:

$\sqrt{34+2}+ 4 = 10$

$\sqrt{36}+ 4 = 10$

$6 + 4 = 10$

$10 = 10$ ✔

Higher - Order Factorable Equation - equation in which zero is isolated and the polynomial on the other side is factorable.

To solve:
1. Isolate zero
2. Factor.
3. Set each factor equal to zero and solve each equation.

Ex: Solve $\mathbf{x^3 = 4x}$

$x^3 = 4x$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x(x + 2)(x - 2) = 0$

$x = 0 \quad x + 2 = 0 \quad x - 2 = 0$

$x = 0$ or $x = -2$ or $x = 2$

MORE SAMPLE QUESTIONS:
More sample questions, including math questions, are available at the following link:

http://media.collegeboard.com/digitalServices/pdf/accuplacer/nc-sample-questions.pdf

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GOOD LUCK ON THE PLACEMENT TEST. WE LOOK FORWARD TO SEEING YOU IN CLASS!

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TUTORING SERVICES

- **ACADEMIC STUDIES CENTER**
  Kenneth B. Hurst
  Continuing Education (CE) Building,
  Room 200
  (910) 938-6259

  REQUIREMENTS:
  - Need to show a government-issued photo ID or a CCCC Student ID card.
  - Must have graduated from high school or completed an Adult High School (AHS) or High School Equivalency (HSE) program.
  - Must take an initial assessment test (TABE test). Results will be used to develop a plan for tutoring service.

- **MATH LAB TUTORING CENTER**
  Math and Science (MS) Building,
  Room 207
  (910) 938-6327

  REQUIREMENTS:
  - Must be a currently enrolled degree-seeking student at Coastal.
  - Need to show your CCCC Coastal Student ID card or present a Math Lab Pass (issued in Student Services).

Notes.